Appendix: definition sheet (3 pages)

Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

M.Sc. in Advanced Computer Science

Automated Reasoning

Thursday 24th January 2008

Time: 09:45 – 11:45

Please answer THREE out of FOUR Questions provided

This is a CLOSED book examination
1. Consider the following Prolog program, which is supposed to compute routes between nodes in a network, given the directed links between pairs of nodes

```prolog
/*1*/ route(X, Y) :- link(X,Y).
/*2*/ route(X, Y) :- route(X, Z), link (Z, Y).
/*3*/ link(f, g).
/*4*/ link(a, b).
/*5*/ link(b, d).
/*6*/ link(c, d).
```

where the nodes are a, b, c, d, f, g. The lines in the program have been labelled for your convenience.

a) Construct part of the SLD-tree representing the execution of the above program, with respect to the goal clause

```
route(X, d).
```

showing all closed branches and describing the structure of any infinite branches.

Show all your working in the construction of the SLD-tree; in particular, each arc should clearly indicate the matching rule and the MGU used within the resolution step. Indicate the order in which the SLD-tree is explored during execution of the Prolog program.

You may wish to abbreviate the predicate names to r and l.

Show how each answer substitution is calculated for closed branches. (16 marks)

b) When searching for all answers, are there any restrictions on the goal clause or alternative search strategies under which the search will terminate? Briefly explain your answer. (4 marks)
2. a) Determine using first order resolution whether or not the following logical consequences hold

i) \[ \forall x(P(x) \rightarrow Q(g(x))) \]
\[ \land \forall x(Q(x) \rightarrow P(g(x))) \]
\[ \land \forall x(P(g(g(x))) \rightarrow P(h(x))) \]
\[ \vdash \forall x(P(x) \rightarrow P(h(x))) \] (8 marks)

ii) \[ \exists y \forall x \forall z(Q(x,z) \rightarrow P(f(x),y)) \]
\[ \land \exists x \exists z(Q(x,z)) \]
\[ \vdash \forall x \exists y P(f(x),y) \] (6 marks)

where \( P, Q \) are predicate symbols, \( x, y, z \) are variables and \( f, g, h \) are function symbols. Show all your working.

b) If a set of clauses \( N' \) is obtained from a set of clauses \( N \) by subsumption deletion, then show that if \( N' \) is satisfied then so is the original set of clauses \( N \).

Subsumption deletion: if \( N \) contains two clauses \( C_1, C_2 \) such that \( C_1 \subseteq C_2 \), then \( N' \) is obtained from \( N \) by deleting \( C_2 \). (6 marks)
3. (Structural transformation, Herbrand interpretations, Redundancy)

a) i) Describe the method of structural transformation for propositional logic. (3 marks)

ii) Give an example to illustrate the method. (3 marks)

iii) Consider this statement:

‘Structural transformation is essential for efficient transformation of formulae into clausal form.’

Referring to this statement, what is the problem with the standard transformation to clausal form? How does structural transformation address this problem? (2 marks)

b) For each set of clauses $N$, write down a Herbrand interpretation $I$ such that $I \models N$. Justify your answer in each case. (5 marks)

i) $N = \{1. \ p(a),
    2. \neg p(a) \lor q(b)\}$

ii) $N = \{1. \ p(a),
    2. \neg p(x) \lor q(b)\}$

iii) $N = \{1. \ p(a),
    2. \neg p(f(x)) \lor p(x)\}$

iv) $N = \{1. \ p(a),
    2. \neg p(x) \lor p(f(x))\}$

Question 3 continues on the following page.
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c) Let the atom ordering be defined by

\[ p > r(a_0) > q(a_1) > q(a_2) > q(a_3) > q(a_4) > q(b) \]

Let \( N \) be the following set of clauses.

1. \( q(a_1) \lor q(a_4) \lor q(b) \)
2. \( q(a_1) \lor q(a_2) \lor q(b) \)
3. \( p \lor q(a_4) \)
4. \( \neg q(a_4) \lor q(a_4) \)
5. \( q(a_3) \lor q(b) \)
6. \( q(a_4) \lor \neg q(a_3) \)

i) For each of the clauses \( C \) below determine whether \( C \) is redundant with respect to \( N \). Give justification for your answers. (4 marks)

A. \( C = r(a_0) \lor \neg r(a_0) \)
B. \( C = q(a_4) \lor p \lor p \)
C. \( C = q(b) \lor q(a_4) \)

ii) Which clauses in \( N \) are redundant with respect to \( N \)? Explain your answer(s). (3 marks)
4. **(Multi-sets, Ordered resolution with selection, Craig Interpolation)**

a) Describe the difference between a multi-set and a set. Illustrate your answer with an example. (2 marks)

b) Let $\succ$ be a total and well-founded ordering on ground atoms such that, if the atom $A$ contains more symbols than $B$ (not counting brackets), then $A \succ B$. Let $N$ be the following set of clauses:

\[
\begin{align*}
1. & \quad \neg q(a) \vee q(f(b)) \\
2. & \quad p(g(y)) \\
3. & \quad q(f(a)) \vee q(g(x)) \\
4. & \quad \neg p(x) \vee \neg q(f(x)) \\
5. & \quad \neg p(f(x)) \vee p(y)
\end{align*}
\]

i) For each clause determine which literals are strictly maximal in it. Give justifications in each case. (7 marks)

ii) Write down which of the clauses in $N$ are resolvable under $\text{Res}^S_{\succ}$ where $S$ is the empty selection function (i.e. no literal is selected). Give the resolvent in each case. (2 marks)

iii) Define a selection function $S$ such that no inference steps are possible under $\text{Res}^S_{\succ}$ on the clauses in $N$. Explain why no inferences are possible. (2 marks)

c) i) State Craig’s interpolation property for propositional logic. (3 marks)

ii) Assume $F$ and $G$ are two propositional formulae and $F \vdash G$ (equivalently $\vdash F \rightarrow G$). Show that, if $F$ and $G$ have no propositional symbols in common, then either $\vdash \neg F$ or $\vdash G$. (4 marks)

END OF EXAMINATION

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