Fundamentals of Parallel and Distributed Systems

Date:   Monday 24th January 2011
Time:  09:45 - 11:45

Please answer any TWO Questions from the Four questions provided

This is an OPEN book examination

The use of electronic calculators is permitted provided they are not programmable and do not store text.
Question 1.

a) It is possible for the following system, described in FSP, to deadlock. Explain how this deadlock occurs and relate it to the four necessary and sufficient conditions for deadlock to occur. (The processes A, B and C may be considered to represent three people, Alice, Bob and Chris).

\[
\begin{align*}
A &= (\text{call.b} \rightarrow \text{wait.c} \rightarrow A).
B &= (\text{call.c} \rightarrow \text{wait.a} \rightarrow B).
C &= (\text{call.a} \rightarrow \text{wait.b} \rightarrow C).
\end{align*}
\]

\[
\mathcal{S} = (A \parallel B \parallel C) /\{\text{call/wait}\}.
\]

[4 marks]

b) The following model attempts to fix the problem by allowing A, B and C to time out from a call attempt. Is a deadlock still possible? If so, describe how the deadlock can occur and give an execution trace leading to the deadlock.

\[
\begin{align*}
A &= (\text{call.b} \rightarrow \text{wait.c} \rightarrow A \mid \text{timeout.a} \rightarrow \text{wait.c} \rightarrow A).
B &= (\text{call.c} \rightarrow \text{wait.a} \rightarrow B \mid \text{timeout.b} \rightarrow \text{wait.a} \rightarrow B).
C &= (\text{call.a} \rightarrow \text{wait.b} \rightarrow C \mid \text{timeout.c} \rightarrow \text{wait.b} \rightarrow C).
\end{align*}
\]

[3 marks]

c) (i) Develop a deadlock free solution, using timeouts, to the problem described in part a). Correct FSP syntax is not required but your description of the proposed solution should be unambiguous in its intended behaviour.

Explain how examining the structure of the LTS of your solution would formally establish its freedom from deadlock.

Informally justify why your revised solution is deadlock free. (An informal description is sufficient because the LTS will almost certainly be too complex to draw manually in an exam.)

[6 marks]

(ii) Is there a fairness issue with your solution? That is, is there a sequence of actions that could be selected to be executed infinitely often by an unfair scheduler, thus preventing the system from making progress (i.e., preventing calls from being made)? If so, give a short example of a sequence of the actions involved.

[4 marks]

d) Develop a deadlock free solution for the problem as stated in part a) that does not include timeouts. State which of the four necessary and sufficient conditions for deadlock you are breaking in your solution. Draw the full LTS and hence show the solution is deadlock free. (The LTS may be as small as three states).

[3 marks]
Question 2.

Consider the following algorithm which implements a 9-point finite difference stencil for the calculation of the solution of a time-dependent problem (as may be found, for example, in a weather modelling application):

\[ U_{i,j} = f(U_{i-1,j}, U_{i-1,j}, U_{i,j}, U_{i+1,j}, U_{i+2,j}, U_{i,j-1}, U_{i,j+1}, U_{i,j+2}) + g_{i,j}, \]

where \( g_{i,j} = g(x_i, y_j) \) is a constant (forcing) term (you may assume period (cyclic) boundary conditions in both \( x \)- and \( y \)-dimensions).

a) Consider a 1-dimensional partition of the \( x \)-dimension of the 2-dimensional solution space into \( P \) blocks (block partition of the \( i \)-index), where \( N \) is an integer multiple of \( P \), so that each task comprises the computation of \( N \times \frac{N}{P} \) components of the solution \( U_{i,j} \).

i) Given that \( t_c \) is the average computation time for \( U_{i,j} \), determine the total computation time of each step of the algorithm and, for \( P \) processors, the corresponding computation time per processor.

[2 marks]

ii) Given that \( t_c \) and \( t_w \) are the start-up cost of a message and the cost per word of transmission (note that the unknowns \( U_{i,j} \) are represented as single precision real variables that occupy one word of memory), derive (and carefully justify) the total communication time for each step of the algorithm and, for \( P \) processors, the corresponding communication time per processor – note that you need to consider separately the cases:

1. \( P = 1, 2, \ldots, \frac{N}{2} \),

and

2. \( P = N \).

[6 marks]

iii) Derive expressions for the relative efficiency of the algorithm.

[2 marks]
b) Consider a 2-dimensional (square) partition of the x- and y-dimension of the 2-dimensional solution space into \( P \) blocks ((square) block partition of the \( i \)- and \( j \)-indices) so that each task again comprises the computation of \( \frac{N^2}{P} \) components of the solution \( U_{i,j} \). Note that you should consider only partitions where the square root of \( P \) is an integer, and assume that \( P \leq \frac{N^2}{4} \).

i) Given that \( t_c \) is the average computation time for \( U_{i,j} \), determine the total computation time of each step of the algorithm and, for \( P \) processors, the corresponding computation time per processor.

[1 mark]

ii) Given that \( t_s \) and \( t_w \) are the start-up cost of a message and the cost per word of transmission, derive the total communication time for each step of the algorithm and, for \( P \) processors, the corresponding communication time per processor.

[4 marks]

iii) Derive an expression for the relative efficiency of the algorithm.

[1 mark]

iv) For the case when \( P \leq \frac{N}{2} \), and \( \sqrt{P} \) is an integer, compare the number of messages and the overall communication costs of the two partition models.

[4 marks]
Question 3.

a) (i) Show that the following FSP processes, S1 and S2, describe the same behaviour. (Hint: This can be achieved by constructing the LTS for the composed process, S1, and for the sequential process, S2, and showing they have the same structure, ignoring possible differences in layout).

\[ P = (a \rightarrow b \rightarrow P). \]
\[ Q = (c \rightarrow b \rightarrow Q). \]
\[ ||S1 = (P||Q). \]
\[ S2 = (a \rightarrow c \rightarrow b \rightarrow S2 \mid c \rightarrow a \rightarrow b \rightarrow S2). \]

[4 marks]

(ii) What would be the observable behaviour of process S1 if the shared action ‘b’ was ‘hidden’ from the environment, as in the following process, S1a below; i.e. what does the ‘minimised’ LTS for S1a look like?

\[ ||S1a = (P||Q) \setminus \{b\}. \]

[3 marks]

b) (i) Give the equivalent sequential process (call it S4) and the LTS for the following composed FSP process, S3, and use this example to explain, informally, how the parallel behaviour of two long running tasks may be modelled in the interleaving model of concurrency employed by FSP. Identify clearly the state (or states) in the resulting LTS that represents the parallel execution of the two processes (A and B).

\[ A = (a.\text{start} \rightarrow a.\text{stop} \rightarrow A). \]
\[ B = (b.\text{start} \rightarrow b.\text{stop} \rightarrow B). \]
\[ ||S3 = (A||B). \]

[6 marks]

(ii) Give a more succinct definition in FSP of process S3 in terms of a single process, P, where P = (start \rightarrow stop \rightarrow P).

[2 marks]

c) Discuss, briefly, in around half a page to a page, the following quote on exascale computing (taken from the International Exascale Software Project Roadmap):

“It is clear that they [exascale platforms] will embody radical changes along a number of different dimensions as compared to the architectures of today’s systems, and that these changes will render obsolete the current software infrastructure for large-scale scientific applications.”

[6 marks]
Question 4.

a) Consider a multi-server queueing system with a single queue and two service points where the arrival rate is 45 customers per hour (and arrivals are assumed to satisfy a Poisson distribution). The average service time is 2 minutes per customer, service times are assumed to be negative-exponentially distributed.
   
i) Calculate the expected waiting time, $W_q$.
   
   [2 marks]

ii) A third service point will be installed when the expected waiting time exceeds 10 minutes. What arrival rate (customers per hour) will trigger the installation of this extra service point?
   
   [4 marks]

b) Consider a queueing system with one service point where the arrival rate is $\lambda$ customers per unit time (and arrivals are assumed to satisfy a Poisson distribution). The average service rate is $\mu$ customers per unit time, service times are assumed to be negative-exponentially distributed, and $\rho = \frac{\lambda}{\mu} < 1$.
   
i) Calculate (in terms of $\rho$) the probability of having to wait for service and the mean queue length.
   
   [2 marks]

ii) In order to reduce the mean queue length, two proposals are under consideration:

   [1] Replace the current service point with a new service point that has twice the capacity (i.e. the new service point has average service rate of $2\mu$ customers per unit time, and service times remain negative-exponentially distributed).

   [2] Add an extra service point with the same capacity (i.e. average service rate $\mu$) as the current service point, with a single queue for both service points.

Show that the mean queue length in case [1] is

$$L_q^{[1]} = \frac{\rho^2}{2(2-\rho)},$$

[3 marks]

and that the mean queue length in case [2] is

$$L_q^{[2]} = \frac{\rho^3}{(2-\rho)(2+\rho)}.$$

[5 marks]

Hence, determine which of the two strategies will lead to the greater reduction in mean queue length?

[4 marks]