Two hours - online hybrid

The exam is hybrid and will be taken online and answered on paper.

EXAM PAPER MUST NOT BE REMOVED FROM THE EXAM ROOM

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Modelling and visualisation of high-dimensional data

Date: Thursday 24th January 2019
Time: 09:45 - 11:45

Please answer All Questions in Section A online
and All Questions in Section B in a separate answerbook

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This is a CLOSED book examination

The use of electronic calculators is permitted provided they are not programmable and do not store text
Section A contains Multiple Choice Questions and is restricted
Section B

Answer all questions from this section.

1. *Singular Value Decomposition* (SVD) is a powerful linear algebraic tool widely used in machine learning. For a $m \times n$ matrix, $X$, describe the SVD and the properties of its components in detail. (6 marks)

2. The *Self-organising map* (SOM) is a popular yet biologically inspired dimension reduction method. Describe the following techniques and terminologies used in the SOM:
   a) What are the *distance metrics* used to measure the difference between a data point and a weight vector and to define a neighbourhood of a neuron, respectively? (2 marks)
   b) What is the *Best-Matching Unit* (BMU)? (2 marks)
   c) What is the *U-matrix*? How is it achieved? (2 marks)

3. Making use of the derivation of the *Principal Component Analysis* algorithm learned from this course unit, specify the utility function especially for obtaining the *second principal component* of a data matrix, $X$, and prove that it is the eigenvector associated with the second largest eigenvalue of $\Sigma$, where $\Sigma$ is the covariance matrix of $X$. (8 marks)