Two hours

UNIVERSITY OF MANCHESTER  
SCHOOL OF COMPUTER SCIENCE

Optimization for learning, planning and problem-solving

Date: Thursday 31st May 2012
Time: 09:45 - 11:45

Please answer ALL Questions from Section A.
Answer TWO Questions from Section B.

Use a SEPARATE answerbook for EACH Section.

For full marks your answers should be concise as well as accurate.
Marks will be awarded for reasoning and method as well as being correct.

This is a CLOSED book examination

The use of electronic calculators is permitted provided they are not programmable and do not store text.

[PTO]
Section A is restricted and cannot be published
Section B

Answer two questions from this Section

21. Branch and Bound

a) Give pseudocode for a generic branch and bound algorithm for a minimization problem. Make sure to include variables for the primal and dual bound and indicate how these are updated. (Briefly explain any procedures called in your pseudocode description).

b) Different stopping criteria can be used with branch and bound. Briefly describe two different stopping criteria suitable for finding an exact (global optimum) solution.

c) Briefly describe two further stopping criteria that can be used to stop Branch and Bound early (i.e. before an optimum has been found) to provide an approximate solution only.

22. Dynamic Programming

a) Explain the single source shortest path problem in weighted graphs. (Use a figure if it would help). Explain how the problem has optimal substructure.

b) Choose any problem involving probabilities that is suitable for dynamic programming and briefly outline it. Explain how a dynamic programming approach for the problem would work. In particular, define the states, stages, constraints, objective, and recurrence (or Bellman equation) you would use. (Either a verbal or symbolic formulation may be used, but marks will be awarded for being succinct and unambiguous).
23. Evolutionary Algorithms

a) Give pseudocode for a generic evolutionary algorithm. For any function or operator included in your pseudocode (such as e.g. crossover), briefly explain what it does. (5 marks)

b) An evolutionary algorithm is being used to solve the Traveling Salesperson Problem. Candidate tours are being represented as permutations of the numbers \{2..N\} to specify the order to visit the cities, after city 1, and before returning to city 1.

   i) State a mutation operator that would typically be used for this problem. Explain how it works and explain why it transmits traits effectively. (2 marks)

   ii) Design a crossover operator that preserves feasibility. More marks will be given if your operator transmits traits effectively from the parents to the offspring. (3 marks)
24. Simulated Annealing

A Job Shop Scheduling optimization problem can be defined as follows.
INSTANCE: A set of jobs $J$. A set of machines $M$. A $|J| \times |M|$ matrix of integer values representing processing times of jobs $j \in J$ on machines $m \in M$.
TASK: Schedule jobs on machines such that at all times machines are processing at most one job, no jobs are interrupted, and the sum of processing times is minimized.

You decide to solve this Problem using simulated annealing. You come up with a representation and an ergodic neighbourhood operator.

You will represent a schedule as a permutation of the jobs. To interpret this as a schedule, the jobs are taken in the order given and processed on the lowest indexed machine that is not currently being used. If all machines are in use, the job is processed on the first machine to be available (taking the lowest indexed machine in the case of ties) immediately that it becomes available.

a) Explain what is meant by the term ergodic when applied to a neighbourhood operator.

   (2 marks)

b) Decide whether or not every optimal solution can be represented by the above representation scheme. Show whether or not this is the case.

   (3 marks)

c) Describe how the parameters of the cooling schedule for the simulated annealer may be selected and calculated. Indicate what information is needed for this.

   (3 marks)

d) Along with the simulated annealer, you decide to test a thresholding accepting algorithm. This algorithm works as simulated annealing but replaces the Boltzmann acceptance function with

   IF $\Delta E > -T$ THEN old configuration:= new configuration

   Explain how, and under what conditions, this might be better than the original simulated annealing algorithm.

   (2 marks)
25. Multiobjective Optimization

a) Define the ‘hypervolume’ indicator. Sketch a proof that the hypervolume of a non-dominated set is at its maximal value for the (true) Pareto front. (5 marks)

b) Consider a multiobjective optimization algorithm that generates points one by one and at every step decides whether or not to store the new point in an archive and/or to delete any other point(s) currently in its archive. Decisions to delete points or not to store them are irreversible. Consider that the algorithm generates a sequence \( S \) of \( k \) points in objective space and the ‘archive’ set must not exceed a maximum cardinality \( M \). Assume \( k > |M| \). DO NOT assume anything about the sequence \( S \), e.g. that points are getting better over time.

i) Give rules for updating the archive that guarantee that at least one of its members is Pareto optimal within the complete sequence \( S \). (2 marks)

ii) Is it possible to give a rule that guarantees that \( M \) will contain all Pareto optima of the complete sequence \( S \), provided \( S \) has fewer than \( M + 1 \) Pareto optima? Explain your reasoning. (3 marks)