Two hours

EXAM PAPER MUST NOT BE REMOVED FROM THE EXAM ROOM AND MUST BE RETURNED

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Optimization for learning, planning and problem-solving

Date: Wednesday 13th May 2015
Time: 14:00 - 16:00

Answer ALL Questions from Section A
Write your answers directly on the exam paper. Only answers written in the boxes on the exam paper will be marked.

Answer TWO Questions from Section B, use a separate answerbook for this Section

This is a CLOSED book examination

The use of electronic calculators is permitted provided they are not programmable and do not store text
Section A contains Multiple Choice Questions and is therefore restricted
21. Branch and Bound

In the degree-constrained minimum spanning tree problem (dMST), you are given a undirected graph \( G = (V,E) \) where \( V \) is a set of vertices and \( E \) is a set of edges connecting every pair of vertices. Also given is a number \( d \leq |V - 1| \) (the degree constraint), and a set of edge costs \( w_{ij} \), representing the cost of connecting vertex \( i \) to vertex \( j \), for all \( i, j \) such that \( i \neq j \). To solve the dMST problem, you must find the minimum cost spanning tree \( (V,E' \subseteq E) \) that contains no vertex with a degree \( > d \).

Note 1: the cost of a tree is the sum of the costs of edges in it.

Note 2: the degree of a vertex is the total number of edges incident to it.

Note 3: a spanning tree has exactly \( |V| - 1 \) edges, and creates a path from every node to every other node without creating any cycles.

You are to design a branch and bound algorithm to solve this problem.

a) Give pseudocode‡ for a generic branch and bound algorithm for a minimization problem. (4 marks)

b) Describe a method to represent a partial dMST solution, and a method for checking the feasibility of such a candidate partial dMST solution. Pseudocode is not necessary, but try to be specific. (3 marks)

c) Describe a function to compute an optimistic bound on a partial dMST solution. (3 marks)

‡ Pseudocode should be human-readable (not machine-readable), precise, and commented to explain any special terms used.
22. Dynamic Programming

A single woman decides to look for a life partner (to settle down with) using an online dating site. It is expensive to use the site, so she decides to go on dates for one month (30 days) only. She will date one new person per day until she has found “the one”, and then she will stop.

You offer to advise her on the best dating strategies, using your knowledge of dynamic programming. She tells you:

- She will be free to date a new person every day for (up to) 30 days
- There are enough people on the dating site, so she will get a date every day
- When she has finished a date, she will either settle down with the person (and stop dating), or she will turn them down, never see them again, and continue dating the next day
- She will be able to work out how much she likes a person on the date. She will be able to rate them on a 1–10 scale, where 10 represents the best quality
- If she dates on day 30, she will settle down with the person no matter their quality
- The quality of a date (from her previous experience) is random, and independent of previous dates. The probability $p(Q)$ of a date (i.e., the person on a date) having a quality $Q$ is given by this table of values:

\[
\begin{align*}
p(1) &= p(2) = p(3) = 0.05 \\
p(4) &= p(5) = 0.15 \\
p(6) &= 0.2 \\
p(7) &= p(8) = 0.15 \\
p(9) &= 0.04 \\
p(10) &= 0.01 
\end{align*}
\]

a) Explain in detail how you would determine the strategy that maximises the expected quality of the life partner the woman settles with. Give the Bellman recurrence relation you would use to calculate the expected values, and explain the decisions she must make, and the rule she should use to make optimal decisions. Full marks will be achieved only for a clear and full explanation. (Actually calculating the expected quality from following the optimal strategy is not needed, however).

(6 marks)

b) You advise that although the strategy you gave her is optimal for maximizing expected value, it might risk settling with a really poor life partner. You offer to show her how to minimize the probability that her final partner has score of less than six instead. Give the Bellman equation that would solve this slightly different version of the problem. This equation encodes a trivial decision rule. What is it? What is the probability under this scheme of still getting a partner of quality less than six?

(4 marks)
23. Evolutionary Algorithms

A Word Cloud is a graphical representation of the words used in a speech, essay or other body of text. An example based on the words in a computer science blog is given below.

Figure 2: An example word cloud

In a Word Cloud, the size of the font used to write a word is proportional to that word’s frequency in the text. However, before the frequencies are worked out, certain common words are excluded.

After this, the words have to be placed on the page, and this is where an optimization algorithm could be used to get a good layout.

Describe a design for an evolutionary algorithm to optimize Word Cloud layouts, including the

- objective function to optimize
- constraint(s)
- solution representation
- initialization procedure
- mutation operator.

Marks will be awarded for precision in description (could it be implemented correctly by following your instructions?) and for any notes about the benefits of choosing a particular design principle. (10 marks)
24. Multiobjective Optimization

   a) Explain how an evolutionary algorithm for multiobjective optimization (finding a Pareto optimal set) may differ from one for single-objective optimization. More marks will be awarded for more differences (or alternative methods) and more precision or detail. Briefly define any terminology you use in your answer.  

   (5 marks)

   b) Give the generic definition of a multiobjective optimization problem.  

   (2 marks)

   c) Give the definition of the Pareto optimal set of solutions to a multiobjective problem, first giving the definition of Pareto dominance on which it relies.  

   (3 marks)