One and a half hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Mathematical Techniques for Computer Science

Monday 21st January 2008
Time: 14:00 – 15:30

Please answer THREE questions from the FOUR provided
Use separate Answerbooks for EACH section
This is a CLOSED book examination

The use of electronic calculators is NOT permitted.
Section A

1. a) Explain the term \textit{normal form} as applied to propositional logic formulae. Describe the procedure to convert a formula into DNF. \quad (6 \text{ marks})

b) Let $A$ be the propositional logic formula

$((p \land q) \Rightarrow r) \Rightarrow ((p \Rightarrow \neg q) \Rightarrow r)$

i) Find a formula in DNF form that is logically equivalent to $A$.

ii) Find a formula in CNF form that is logically equivalent to $A$. \quad (14 \text{ marks})

2. a) Explain what is meant by the term \textit{formal language}, illustrating your answer with examples. \quad (4 \text{ marks})

b) A small formal language $L$ of \textit{expressions} is defined by the rules:-

- Each of the lower case letter symbols $a \ldots z$ is an expression
- If $A$ is an expression, then so is $A^*$
- If $A$ is an expression, then so is $A^#$
- If $A$ and $B$ are expressions, then so are $AB\Delta$, $AB\triangledown$ and $AB\bigcirc$

i) Give examples of expressions in this language, giving the parse tree for each. \quad (6 \text{ marks})

ii) Show that the following is an expression in $R$ by giving its parse tree:

$sr \Delta^* p\#q * \bigcirc tu * \Delta\triangledown$ \quad (10 \text{ marks})
3.  a) Given two events $A$ and $B$, give formulae for the conditional probabilities $\Pr(B \mid A)$ and $\Pr(A \mid B)$.  

b) The following facts are known regarding two events $A$ and $B$:

$$\Pr(A \cup B) = \frac{2}{3}, \quad \Pr(A \cap B) = \frac{1}{3}, \quad \Pr(A \mid B) = \frac{1}{2}.$$ 

Find the following:

i) $\Pr(B);$  
ii) $\Pr(A);$  
iii) $\Pr(B \mid A);$  
iv) $\Pr(A \cap B^C);$  
v) $\Pr(A^C \cap B).$  

4.  a) State one condition that will ensure independence of two events $A$ and $B$.  

b) If $A$ and $B$ are two independent events show that $A$ and $B^C$ are also independent events.  

c) A town has two fire engines operating independently. The probability that a specific fire engine is available when needed is 0.95. Find the following probabilities:

i) Probability that neither is available when needed.  
ii) Probability that both are available when needed.  
iii) Probability that a fire engine is available when needed.