Theory of Games and Game Models

Thursday 24th January 2008     Time: 14:00 – 16:00

Please answer any THREE Questions from the FIVE questions provided

This is a CLOSED book examination

The use of electronic calculators is NOT permitted
1. Two players play the following version of Nim: The player who takes the last match wins, and at the start there is one pile with one match, and one pile with two matches. The first player hides the piles from view. He then makes a move according to the rules of Nim (that is, he takes any positive number of matches from the same pile). He then tells the other player how many matches are left in play.

The second player now has to decide how many matches she will remove on her first move, still without seeing the current state of play. For this move only, she is allowed to remove no match at all, so she can announce that she will remove 0, 1, or 2 matches. Player 1 now unveils the piles of matches. If it turns out that Player 2 cannot carry out a legal move in accordance with her announcement she loses two pounds to player 1; if she wins on this move, she wins two pounds from Player 1.

If the game has not yet been decided they keep playing Nim, now according to the usual rules. The winner gets one pound from the other.

a) Draw a game tree for this game. (5 marks)

b) Describe the strategies for both players. To slightly reduce the number of strategies you are allowed to do the following: If there is a move that immediately wins, you may assume that the strategy takes it. (5 marks)

c) Give the matrix form of the game. (3 marks)

d) Find an equilibrium point for the game. (4 marks)

e) What is the value of the game? Is it fair? How would you play as Player 1, how as Player 2? (3 marks)
2. a) Find all pure strategy equilibrium points for this game and give its value. (5 marks)

\[
\begin{pmatrix}
2 & -1 & 0 & -1 & 0 \\
0 & -2 & 1 & -1 & 0 \\
1 & -1 & -1 & -2 & 1 \\
-1 & -1 & 0 & -1 & -1 \\
-2 & -1 & 1 & -1 & -2 \\
\end{pmatrix}
\]

b) Find as many additional equilibrium points as you can for the game from (a). (3 marks)

c) Find an equilibrium point and the value for the following matrix game. (4 marks)

\[
\begin{pmatrix}
-3 & 2 \\
6 & -1 \\
\end{pmatrix}
\]

d) Find one equilibrium point and the value for the following matrix game. (8 marks)

\[
\begin{pmatrix}
1 & -1 & -2 & 0 \\
0 & -1 & -1 & 0 \\
-2 & 0 & 1 & -1 \\
-1 & -1 & 0 & -1 \\
\end{pmatrix}
\]

3. Consider the following matrix game:

\[
\begin{pmatrix}
0 & -1 & 3 \\
3 & 5 & -3 \\
\end{pmatrix}
\]

Find all the equilibrium points for this game, and prove that you have done so. [Hint: First look at Player 1. Do this by using the techniques for graphically solving games.] (20 marks)
4. a) What are the main components of a program that can play a game such as chess or go? Give a short description of each. (9 marks)

b) How does such a program decide which move to make, and what role do the components identified in (a) play in that process? (6 marks)

c) Compare the approach outlined in (b) to a person playing such a game. What are the main differences? How do human beings fare when playing against a computer? (5 marks)

5. a) Describe the general Prisoners' Dilemma (PD) game. (4 marks)

b) What are the equilibrium points for:
   - the general PD game and
   - for five rounds of the general PD game? (5 marks)

c) What can you say about the equilibrium points of the PD game played:
   - a fixed number of rounds and
   - an indefinite number of rounds? (6 marks)

d) Show that AlwaysD is not necessarily the best strategy when playing the indefinitely repeated PD game. (5 marks)

END OF EXAMINATION