The following equations may be used without proof.

\[ i^{-1}T_i = \begin{bmatrix} C_{0_i} & -C_{a_i}S_{0_i} & S_{a_i}S_{0_i} & A_iC_{0_i} \\ S_{0_i} & C_{a_i}C_{0_i} & -S_{a_i}C_{0_i} & A_iS_{0_i} \\ 0 & S_{a_i} & C_{a_i} & D_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

General Homogeneous Transformation matrix

### Outward recursions

1. Angular velocity of link: \( \dot{\omega}_i = \omega_{i-1} + \dot{\theta}_i \)

2. Angular acceleration of link \( \ddot{\omega}_i = \dot{\omega}_{i-1} + \ddot{\theta}_i \)

3. Linear acceleration of link at frame origin \( \dot{i}_i = \dot{i}_{i-1} + L_i \left[ -\omega_i^2 \right] \)

4. Linear acceleration of link at centroid \( i_{\dot{a}_i} = \dot{i}_{i-1} + c_i \left[ -\omega_i^2 \right] \)

5. Resultant force acting on link at centroid \( i_F = m_i \dot{i}_{i-1} \)

6. Resultant moment acting on link around centroid \( N_i = I_i \dot{\theta}_i \)

### Inward recursions

1. Force exerted on link \( i \) by link \( i-1 \)

\[ \text{if} \quad i_{F_{i+1}} = i_{R_{i+1}} + i_{F_{i+1}} \quad \text{if} \quad i_{F_{i+1}} = i_{F_i} + i_{F_{i+1}} \]

2. Torque exerted on link \( i \) by link \( i-1 \)

\[ \tau_i = N_i + \tau_{i+1} + i_{F_{i+1}}c_i + i_{F_{i+1}}(L_i - c_i) \]

### Standard Equations for the Newton-Euler method

\[ \text{if} \quad T = \begin{bmatrix} R & P \\ 0 & 1 \end{bmatrix} \quad \text{then} \quad T^{-1} = \begin{bmatrix} R^T & -R^TP \\ 0 & 1 \end{bmatrix} \]

Matrix inversion formula

\[ \frac{\dot{V}^2}{a} - tV + \theta = 0 \]

Linear trajectory with parabolic blends

\[ R = \frac{L}{2} \left( \frac{V_r + V_i}{V_r - V_i} \right) \]

Instantaneous centre of rotation (ICR) radius

\[ Q_{i+1}(s, a) \leftarrow (1 - \beta)Q_i(s, a) + \beta(R(s_{i+1}) + \gamma \max_a Q_i(s_{i+1}, a)) \]

Q-learning equation
1. a) In the context of the Denavit-Hartenberg notation, explain what is known as the link perpendicular. (2 marks)

b) For each case shown in figure 1 below, draw the link perpendicular line and explain any peculiarities in where the line can be drawn. (6 marks)

![Figure 1: Illustrations of relative configurations of two joints](image)

**Figure 1** Illustrations of relative configurations of two joints

c) Figure 2 shows a serial link manipulator. Redraw the manipulator using Denavit-Hartenburg convention, ensuring all frame axes are fully labelled. (6 marks)

![Figure 2: Serial link Manipulator](image)

**Figure 2** Serial link Manipulator

(Question 1 continues on the following page)
d) Form the homogeneous transformation matrix $^A T_B$ that results from a rotation matrix $R$ and translation $P$ as below:

$$
R = \begin{bmatrix}
1 & 0 & 0 \\
0 & 0.87 & -0.5 \\
0 & 0.5 & 0.866
\end{bmatrix}, \quad P = \begin{bmatrix}
1 \\
3 \\
5
\end{bmatrix}
$$

Hence find $^B Q$ if $^A Q = [2, 4, 5]^T$. Show all the steps in your calculations.

(6 marks)
2. a) Explain the meaning of the terms $\omega$ & $\dot{\theta}$ in the Newton-Euler inverse dynamics method, and how they can have different values on the same link. (4 marks)

b) A single leg of a multi-legged robot is shown in figure 3. The physical parameters of the leg are shown in table 1. This leg is supporting the weight of the robot by applying a force of 100N vertically to the ground, as shown on the diagram. The robot is stationary. Use the Newton-Euler inverse dynamics method to calculate the joint 1 and 2 torques. Briefly comment on how the torques calculated intuitively correspond to the leg configuration. (12 marks)

![Figure 3: Robot leg](image)

<table>
<thead>
<tr>
<th></th>
<th>Link 1</th>
<th>Link 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>10 kg</td>
<td>15 kg</td>
</tr>
<tr>
<td>Moment of inertia about centroid</td>
<td>4 kgm²</td>
<td>3 kgm²</td>
</tr>
<tr>
<td>Joint angular velocity</td>
<td>0 rad/s</td>
<td>0 rad/s</td>
</tr>
<tr>
<td>Joint angular acceleration</td>
<td>0 rad/s²</td>
<td>0 rad/s²</td>
</tr>
<tr>
<td>Link length</td>
<td>0.45m</td>
<td>0.45m</td>
</tr>
<tr>
<td>Centroid location (c₁)</td>
<td>0.25m</td>
<td>0.2m</td>
</tr>
</tbody>
</table>

Table 1 Parameter values

c) The robot leg torques are to be generated by permanent magnet electric DC motors without gearboxes. For joint 1, an electric motor with a torque constant ($k_t$) of 10 has been chosen. In the static case described in part 2b, what current will the motor draw? Comment on whether this actuator type and characteristics are a suitable choice for this joint. (4 marks)
3. a) Describe trajectory planning in the context of robotic systems. (2 marks)

A revolute manipulator joint is to move from $10^0$ to $60^0$. The maximum possible velocity the joint can achieve is $0.5 \text{ rad/s}$ and the acceleration is $3 \text{ rad/s}^2$

b) Calculate the time in which the movement can be achieved if a linear trajectory with parabolic blends is used and sketch a graph of this trajectory. (8 marks)

c) In the context of trajectory planning, describe what is known by ‘jerk’ and how the polynomial trajectory technique minimises the peak jerk value. Use a sketch to illustrate your answer. (5 marks)

d) The revolute joint is to be accurately simulated. What real world factors need to be included in simulation to make it realistic? (5 marks)

4. A mobile robot is to be developed for precision movements within a nuclear power plant. The robot will drive over flat floor as it moves around the plant. The robot is required to collect a full container from one location and drop it off in another location within the building.

a) Describe and sketch an appropriate mobile robot considering the required sensors, wheel or leg configuration and actuators. (10 marks)

b) What kind of navigation system (map building and following) would you recommend for the robot and why? (2 marks)

c) Explain why most maps built using odometry suffer from noise and how this affects the maps produced. (3 marks)

d) Describe a method to periodically correct for the errors caused when using odometry as a primary sensor and briefly explain how it could be implemented on the robot. Other sensors can be added to the robot. (5 marks)

5. a) In the context of autonomous robots, briefly explain the concepts of supervised learning, reinforcement learning and unsupervised learning; without reference to specific implementations of each technique. (6 marks)

b) A wheeled robot has been designed and built to drive between locations avoiding obstacles. The robot has a ring of sonar sensors, GPS, bumpers and some manual controls for human input. Describe how 2 methods of learning obstacle avoidance could be implemented on the robot, and the advantages/disadvantages of both. (10 marks)

c) Is it appropriate to apply learning to a jointed manipulator working within a factory environment? Describe the benefits and drawbacks of learning in this application. (4 marks)

END OF EXAMINATION