Closed Book Examination

COMP10020

One and a half hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Mathematical Techniques for Computer Science

Monday 26th January 2009

Time: 09:45 – 11.15

Please answer THREE questions from the FOUR provided

Use separate Answerbooks for EACH section

The use of electronic calculators is NOT permitted.
Section A

1. a) Explain the term *normal form* as applied to propositional logic formulae. Describe the procedure to convert a formula into DNF. (6 marks)

   b) i) Show that \((a \land b) \lor \neg a\) is logically equivalent to \(\neg a \lor b\). (4 marks)

   ii) Let \(A\) be the propositional logic formula

   \[
   ((x \Rightarrow z) \Rightarrow ((x \land y) \Rightarrow z))
   \]

   By expressing \(A\) in DNF and simplifying the resulting expression, show that \(A\) is a tautology. [Hint: you may find the result from i) useful in your proof.] (10 marks)

2. a) Explain what is meant by the term *formal language* and state examples of two formal languages. (4 marks)

   b) A small formal language \(L\) of *expressions* is defined by the rules:-

   - Each of the lower case letter symbols \(a\) ... \(z\) is a expression
   - If \(A\) is a expression, then so is \(\neg A\)
   - If \(A\) and \(B\) are expressions, then so are \(+AB\), \(*AB\) and \(/AB\)

   i) Give three examples of expressions in this language, giving the parse tree for each. (6 marks)

   ii) Show that the following is an expression in \(L\) by giving its parse tree:

   \[
   / \ - \ * \ x \ - \ y \ - \ /w \ * \ zv \ + \ xy
   \]
Section B

3. a) Describe what is meant by a binomial experiment. (2 marks)
   
   b) Let $X$ denote a binomial random variable with parameters $m$ and $p$.
      
      i) Write in full the probability mass function of $X$. (5 marks)
      
      ii) What is the expected value of $X$? (2 marks)
      
      iii) What is the variance of $X$? (2 marks)

   c) In the past, two building contractors, A and B, have competed for 50 contracts. A won 10 and B won 40 of these contracts. The contractors have both been asked to tender for four new contracts. On the basis of their past performance, what is the probability that
      
      i) Contractor A will win all the contracts? (2 marks)
      
      ii) Contractor B will win at least one contract? (5 marks)
      
      iii) Contractor A will win exactly two contracts? (2 marks)

4. a) State a condition that will ensure independence of two events $A$ and $B$. (2 marks)
   
   b) If $A$ and $B$ are two independent events show that
      
      i) $A$ and $B^C$ are also independent events. (3 marks)
      
      ii) $A^C$ and $B$ are also independent events. (3 marks)
      
      iii) $A^C$ and $B^C$ are also independent events. (3 marks)

   c) Consider an experiment involving two successive rolls of a 4—sided die in which all 16 possible outcomes are equally likely and have probability 1/16.
      
      i) Are the events $A = \{1st\ roll\ results\ in\ 1\}$ and $B = \{2nd\ roll\ results\ in\ 2\}$ independent? (3 marks)
      
      ii) Are the events $A = \{1st\ roll\ is\ a\ 1\}$ and $B = \{sum\ of\ the\ two\ rolls\ is\ a\ 5\}$ independent? (3 marks)
      
      iii) Are the events $A = \{maximum\ of\ the\ two\ rolls\ is\ 2\}$ and $B = \{minimum\ of\ the\ two\ rolls\ is\ 2\}$ independent? (3 marks)

END OF EXAMINATION