Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Understanding Programming Languages

Thursday 22nd January 2009
Time: 14:00 – 16:00

Please answer any THREE Questions from the FIVE questions provided

The use of electronic calculators is NOT permitted.
All questions on this paper refer to the DO language
whose syntax and structural operational semantics are given as follows.

A Denotational Semantics for Do

\[
\begin{align*}
  a & ::= \; n \mid x \mid a_1 + a_2 \mid a_1 \times a_2 \mid a_1 - a_2 \\
  b & ::= \; \text{true} \mid \text{false} \; | \; a_1 = a_2 \mid a_1 \leq a_2 \mid \neg b \; | \; b_1 \land b_2 \\
  S & ::= \; x ::= a \mid \text{skip} \mid S_1 ; S_2 \mid \text{if } b \text{ then } S_1 \text{ else } S_2 \mid \text{do } S \text{ while } b
\end{align*}
\]

Table 1: Syntax of Do

<table>
<thead>
<tr>
<th>Syntax</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{do}[x ::= a]$</td>
<td>$s[x \mapsto A[a] \cdot s]$</td>
</tr>
<tr>
<td>$S_{do}[\text{skip}]$</td>
<td>$\text{id}$</td>
</tr>
<tr>
<td>$S_{do}[S_1 ; S_2]$</td>
<td>$S_{do}[S_2] \circ S_{do}[S_1]$</td>
</tr>
<tr>
<td>$S_{do}[\text{if } b \text{ then } S_1 \text{ else } S_2]$</td>
<td>$\text{cond}(B[b], S_{do}[S_1], S_{do}[S_2])$</td>
</tr>
<tr>
<td>$S_{do}[\text{do } S \text{ while } b]$</td>
<td>$\text{FIX } F \text{ where } F \cdot g = \text{cond}(B[b], g, \text{id}) \circ S_{do}[S]$</td>
</tr>
</tbody>
</table>

Table 2: Denotational Semantics for Do

<table>
<thead>
<tr>
<th>Syntax</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B[\text{true}]$</td>
<td>$\text{tt}$</td>
</tr>
<tr>
<td>$B[\text{false}]$</td>
<td>$\text{ff}$</td>
</tr>
</tbody>
</table>
| $B[a_1 = a_2]$ | \[
\begin{cases}
  \text{tt} \text{ if } A[a_1] \cdot s = A[a_2] \cdot s \\
  \text{ff} \text{ if } A[a_1] \cdot s \neq A[a_2] \cdot s \\
\end{cases}
\]
| $B[a_1 \leq a_2]$ | \[
\begin{cases}
  \text{tt} \text{ if } A[a_1] \cdot s \leq A[a_2] \cdot s \\
  \text{ff} \text{ if } A[a_1] \cdot s > A[a_2] \cdot s \\
\end{cases}
\]
| $B[\neg b]$ | \[
\begin{cases}
  \text{tt} \text{ if } B[b] \cdot s = \text{ff} \\
  \text{ff} \text{ if } B[b] \cdot s = \text{tt} \\
\end{cases}
\]
| $B[b_1 \land b_2]$ | \[
\begin{cases}
  \text{tt} \text{ if } B[b_1] \cdot s \text{ and } B[b_2] \cdot s \\
  \text{ff} \text{ if not } (B[b_1] \cdot s \text{ and } B[b_2] \cdot s) \\
\end{cases}
\]

Table 3: The Semantics of Boolean Expressions

<table>
<thead>
<tr>
<th>Syntax</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A[n]$</td>
<td>$N[n]$</td>
</tr>
<tr>
<td>$A[x]$</td>
<td>$s \cdot x$</td>
</tr>
<tr>
<td>$A[a_1 + a_2]$</td>
<td>$A[a_1] \cdot s + A[a_2] \cdot s$</td>
</tr>
<tr>
<td>$A[a_1 \times a_2]$</td>
<td>$A[a_1] \cdot s \times A[a_2] \cdot s$</td>
</tr>
<tr>
<td>$A[a_1 - a_2]$</td>
<td>$A[a_1] \cdot s - A[a_2] \cdot s$</td>
</tr>
</tbody>
</table>

Table 4: The Semantics of Arithmetic Expressions
1. a) Give a natural semantics for the statement part of the Do language. (5 marks)
   b) Prove that your natural semantics is equivalent to the denotational semantics of Do. (10 marks)
   c) Extend your natural semantics to handle abort statements. (5 marks)

2. a) Give a structural operational semantics for the statement part of the Do language. (5 marks)
   b) Prove that your structural operational semantics is equivalent to the denotational semantics of Do. (10 marks)
   c) Extend your structural operational semantics to handle statements of the form:
      \[ S_1 \text{ par } S_2 \] (5 marks)

3. a) Give an abstract machine and compiler for the statement part of the Do language. (5 marks)
   b) Prove that for the statement part of the language your compiler and abstract machine is equivalent to the denotational semantics of Do. (10 marks)
   c) Extend your abstract machine and compiler to handle non-deterministic or statements of the form:
      \[ S_1 \text{ or } S_2 \] (5 marks)
4. a) Give a continuation semantics for the statement part of the Do language. (5 marks)

b) Prove that your continuation semantics is equivalent to the denotational semantics of Do. (10 marks)

c) Extend your continuation semantics to handle repeat statements. (5 marks)

5. a) Give an axiomatic semantics for partial correctness for the statement part of the Do language. (5 marks)

b) Prove that your axiomatic semantics is sound and complete with respect to the denotational semantics of Do. (10 marks)

c) An axiomatic semantics for total correctness would encompass the termination properties; if \([P] S [Q]\) holds, then whenever \(P\) is a precondition, we would know that \(S\) terminates, and the post-condition \(Q\) holds. What problems occur when the axiomatic semantics is for total correctness? (5 marks)

END OF EXAMINATION