Theory of Games and Game Models

Monday 19th January 2009     Time: 09:45 – 11:45

Please answer any THREE Questions from the FIVE questions provided

The use of electronic calculators is NOT permitted
1. Player 1 holds three cards, two of which are red and one of which is black. He can see these cards, but his opponent cannot. He discards one of the cards without showing it to the other player and puts the others face down on the table. He then ‘calls’ by announcing either ‘two reds’ or ‘black/red’. ‘black/red’ counts as better the ‘two reds’. Player 2 now has the choice to either ‘accept’ or to ‘question’ this call. Once she has done that the cards are turned over. The pay-offs are defined as follows.

- If Player 1 calls the correct hand and Player 2 accepts Player 1 wins one unit;
- If Player 1 calls the hand better than it is and Player 2 accepts Player 1 wins three units;
- If Player 1 calls the hand worse than it is and Player 2 accepts Player 2 wins three units;
- If Player 2 questions the call then the above pay-offs are doubled and reversed.

a) Draw a game tree for this game. (4 marks)
b) Describe all strategies for both players. (4 marks)
c) Give the matrix form of the game. (4 marks)
d) Determine an equilibrium point of the game. (5 marks)
e) What is the value of the game? Would you consider the game fair? Describe in terms of the original game how the two players should behave. (3 marks)
2. a) Find all pure strategy equilibrium points for this game and give its value. 

\[
\begin{array}{ccccc}
2 & 1 & 1 & 0 & 2 \\
1 & 1 & 2 & 1 & 1 \\
-1 & 1 & 2 & 1 & 1 \\
1 & 1 & -1 & 1 & 1 \\
1 & 1 & 2 & 1 & 1 \\
\end{array}
\]

(5 marks)

b) Find an equilibrium point for the following game. 

\[
\begin{array}{cccc}
-4 & -3 & -2 & -1 \\
-3 & -2 & -3 & -3 \\
-3 & -5 & 0 & -1 \\
-2 & 0 & -5 & -4 \\
\end{array}
\]

(8 marks)

c) Find an equilibrium point and the value for the following matrix game. (4 marks)

\[
\begin{array}{cc}
3 & -1 \\
-2 & 5 \\
\end{array}
\]

d) Show that \((1/4, 1/4, 1/2), (1/4, 1/2, 1/4)\) is not an equilibrium point for Paper-Stone-Scissors. (3 marks)

3. Consider a game for 2 players where each player has two strategies, and assume that the pay-offs for one player are the negatives of the pay-offs for the other. Prove that this game has at least one equilibrium point, which may be either pure or mixed. (20 marks)

4. a) Pick an example of a game tree and demonstrate how the minimax algorithm works for it. Describe in a few sentences how the algorithm operates. What does the algorithm calculate in a 2-person zero-sum game? What does it calculate for games with more than two players which are not necessarily zero-sum? (9 marks)

b) Pick an example and show how the alpha-beta algorithm can carry out the same calculation as the minimax algorithm without having to traverse the entire game tree. (5 marks)

c) How is the alpha-beta algorithm used by a typical game-playing program? Describe briefly how such a program determines which move to play next, and what role the alpha-beta algorithm plays in that process. (6 marks)
5.  
   a) Describe the indefinitely repeated Prisoner’s Dilemma game. (6 marks)  
   
   b) When do we say that a strategy A can be invaded by strategy B? What is the  
      connection between this definition and an invasion in an indefinitely repeated  
      Prisoner’s Dilemma game? (4 marks)  
   
   c) Describe an indefinitely repeated Prisoner’s Dilemma game in which the  
      GRUDGE strategy can be invaded by giving all pay-offs and a probability. Prove  
      your claim. (6 marks)  
   
   d) Can you think of a situation in real life that is like playing the Prisoner’s  
      Dilemma game, but with more than two players? In which sense is your situation  
      similar to playing that game? (4 marks)