Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Understanding Programming Languages

Date: Wednesday 27\textsuperscript{th} January 2010

Time: 09.45 – 11.45

Please answer any THREE Questions from the FIVE questions provided

The use of electronic calculators is NOT permitted.
All questions on this paper refer to the While language whose syntax and axiomatic semantics are given as follows.

\[
a ::= n \mid x \mid a_1 + a_2 \mid a_1 \times a_2 \mid a_1 - a_2 \\
b ::= \text{true} \mid \text{false} \mid a_1 = a_2 \mid a_1 \leq a_2 \mid \neg b \mid b_1 \land b_2 \\
S ::= x := a \mid \text{skip} \mid S_1 ; S_2 \mid \text{if} \ b \ \text{then} \ S_1 \ \text{else} \ S_2 \mid \text{while} \ b \ \text{do} \ S
\]

Table 1: Syntax of While

- \[\text{[asg]}\] \{ \ P[x \leftarrow A[a]] \} \ x := a \{ P \}
- \[\text{[skip]}\] \{ P \} \text{skip} \{ P \}
- \[\text{[comp]}\] \{ P \} S_1 \{ Q \}, \{ Q \} S_2 \{ R \} \\
\{ P \} S_1 ; S_2 \{ R \}
- \[\text{[if]}\] \{ B[b] \land P \} S_1 \{ Q \}, \{ \neg B[b] \land P \} S_2 \{ Q \} \\
\{ P \} \text{if} \ b \ \text{then} \ S_1 \ \text{else} \ S_2 \{ Q \}
- \[\text{[while]}\] \{ B[b] \land P \} S \{ P \} \\
\{ P \} \text{while} \ b \ \text{do} \ S \{ \neg B[b] \land P \}
- \[\text{[cons]}\] \{ P' \} S \{ Q' \} \\
\{ P \} S \{ Q \} \text{ when } P \Rightarrow P' \text{ and } Q' \Rightarrow Q

Table 2: Axiomatic System for Partial Correctness of While

\[
B[\text{true}] s = \text{tt} \\
B[\text{false}] s = \text{ff} \\
B[a_1 = a_2] s = \begin{cases} \text{tt} & \text{if } A[a_1] s = A[a_2] s \\ \text{ff} & \text{if } A[a_1] s \neq A[a_2] s \end{cases} \\
B[a_1 \leq a_2] s = \begin{cases} \text{tt} & \text{if } A[a_1] s \leq A[a_2] s \\ \text{ff} & \text{if } A[a_1] s > A[a_2] s \end{cases} \\
B[\neg b] s = \begin{cases} \text{tt} & \text{if } B[b] s = \text{ff} \\ \text{ff} & \text{if } B[b] s = \text{tt} \end{cases} \\
B[b_1 \land b_2] s = \begin{cases} \text{tt} & \text{if } B[b_1] s \text{ and } B[b_2] s \\ \text{ff} & \text{if } \text{not} (B[b_1] s \text{ and } B[b_2] s) \end{cases}
\]

Table 3: The Semantics of Boolean Expressions

\[
A[n] s = N[n] \\
A[x] s = s x \\
A[a_1 + a_2] s = A[a_1] s + A[a_2] s \\
A[a_1 \times a_2] s = A[a_1] s \times A[a_2] s \\
A[a_1 - a_2] s = A[a_1] s - A[a_2] s
\]
1. a) Give a non-trivial natural semantics for the statement part of the While language. (5 marks)
   b) Prove that the axiomatic semantics of While is sound and complete with respect to your natural semantics. (10 marks)
   c) Extend your natural semantics to handle procedure definitions with static scope for both variables and procedures. (5 marks)

2. a) Give a non-trivial structural operational semantics for the statement part of the While language. (5 marks)
   b) Prove that the axiomatic semantics of While is sound and complete with respect to your structural operational semantics. (10 marks)
   c) Extend your structural operational semantics to handle statements of the form:
      $$\text{for } x := a_1 \text{ to } a_2 \text{ do } S$$ (5 marks)

3. a) Give a non-trivial abstract machine and compiler for the statement part of the While language. (5 marks)
   b) Prove that for the statement part of the language the axiomatic semantics of While is sound and complete with respect to your compiler and abstract machine. (10 marks)
   c) Extend your abstract machine and compiler to handle statements of the form:
      $$\text{do } S \text{ while } b$$ (5 marks)
4. a) Give a non-trivial denotational semantics for the statement part of the **While** language. (5 marks)

   b) Prove that the axiomatic semantics of **While** is sound and complete with respect to your denotational semantics. (10 marks)

   c) Either extend your denotational semantics to handle non-deterministic **par** statements or explain why this is not possible. (5 marks)

5. a) Give an axiomatic semantics for partial correctness for **abort** statement. (5 marks)

   b) Give an axiomatic semantics for partial correctness for **repeat** statements. (5 marks)

   c) Give an axiomatic semantics for partial correctness for **for** statements (5 marks)

   d) Give an axiomatic semantics for partial correctness for non-deterministic **or** statements (5 marks)