Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Logic and Modelling

Date: Tuesday 26th January 2016

Time: 09:45 - 11:45

Please answer any THREE Questions from the FOUR Questions provided

This is a CLOSED book examination

The use of electronic calculators is NOT permitted

1.

a) Apply the splitting algorithm to check whether the following formula is satisfiable.

$$\neg((p \to (q \to r)) \leftrightarrow ((p \to \neg r) \to (p \to \neg q)))$$

Split on the variable p first. Show the splitting tree. (6 marks)

b) Convert the formula $p \wedge q \leftrightarrow \neg p \to \neg q$ into clausal normal form using the definitional clausal form transformation. (5 marks)

c) Draw two OBDDs for the formula $(q \to r) \to \neg q$ corresponding to two variable orders: $q > r$ and $r > q$. (4 marks)

d) Consider propositional variables $p_1, p_2, \ldots, p_n$, where $n \geq 3$. Formalise the following statements using propositional formulas:

- At least two variables are true among $p_1, p_2, \ldots, p_n$. (2 marks)
- At most two variables are true among $p_1, p_2, \ldots, p_n$. (2 marks)
- Exactly two variables are true among $p_1, p_2, \ldots, p_n$. (1 marks)

If you are using the T notation from the lectures you should explicitly define it.

2.

a) Consider the Walk SAT algorithm (WSAT).

- Explain briefly how the WSAT algorithm selects a variable for flipping.

(4 marks)

- Consider the following set of clauses:

$$\begin{array}{c} p_0 \lor \neg p_1 \lor p_2 \\ \neg p_0 \lor \neg p_1 \lor p_3 \\ \neg p_0 \lor p_2 \end{array}$$

Compute the probabilities of flipping for each of the variables p_0, p_1, p_2, p_3 at the first step of the WSAT algorithm applied to this set of clauses and the initial interpretation $\{p_0 \mapsto 1, p_1 \mapsto 1, p_2 \mapsto 0, p_3 \mapsto 0\}$. (4 marks)

b) Transform the following formula into the prenex normal form. (6 marks)

$$\forall q(\forall p(p \to \exists q(q \lor \neg p)) \to \forall p(\neg q \to p)).$$

c) Write down LTL formulas expressing the following properties:

i Formulas A and B never hold at the same state. (2 marks)

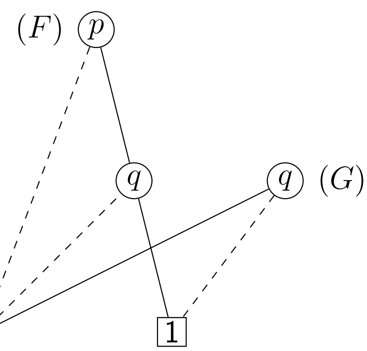
ii Formula A holds only finitely often; (2 marks)

iii Forumla A holds at most at one state. (2 marks)

3.

a) Consider the following global dag D containing two nodes representing OBDDs for some formulas F and G. Build OBDDs which represent formulas $F \vee G$ and $F \wedge G$ and integrate them into D. Only draw the resulting DAG. Show nodes in the new DAG which represent $F \vee G$ and $F \wedge G$.

(8 marks)



b) A variable x in propositional logic of finite domains has the domain $\{b, e, d\}$. Using the tableau method, check whether the formula

$$\neg \mathsf{x} \in \{b, d\} \rightarrow \mathsf{x} \in \{d, e\} \wedge \mathsf{x} \in \{e, b\}$$

is valid. Explain your answer.

(9 marks)

c) Let A be a propositional formula over variables $p_1, \ldots, p_{n-1}$. What is the number of models of the formula $p_n \leftrightarrow A$? Explain your answer. (3 marks)

4.

a) Consider the following QBF formula in CNF (12 marks)

$$\boxed{\begin{array}{c} \forall p \exists q \forall r \\ p \vee q \vee r \\ p \vee \neg q \vee \neg r \\ p \vee q \vee \neg r \\ \neg p \vee q \vee r \\ \neg p \vee q \vee \neg r \end{array}}$$

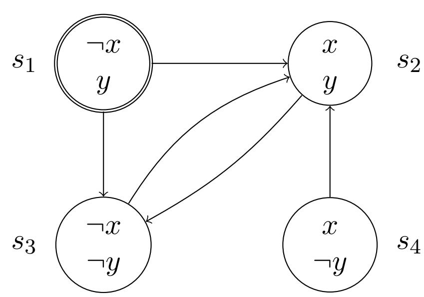
Evaluate this formula using the DLL algorithm. Show all steps of the algorithm. Is this formula true or false?

b) Consider the transition system with the state transition graph shown below.

i Find a symbolic representation of the set of states reachable from the inital state. (2 marks)

ii Find a symbolic representation of the set of transitions $\{(s_1, s_2), (s_1, s_3)\}$. (2 marks)

iii Find a symbolic representation of the set of states unreachable from s_2. (2 marks)



c) Explain briefly how validity and equivalence of propositional formulas can be expressed in terms of satisfiability. (2 marks)

END OF EXAMINATION