COMP36111

Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Advanced Algorithms I

Date: Wednesday 27th January 2016

Time: 14:00 - 16:00

Please answer any THREE Questions from the FOUR Questions provided

Use a SEPARATE answer book for each SECTION.

This is a CLOSED book examination

The use of electronic calculators is permitted provided they are not programmable and do not store text

Section A

Use separate answer books for the two sections.

1. a) Define what is meant by both the terms *undirected graph* and *directed graph*.
(2 marks)

   b) Describe an algorithm for the depth-first search of directed graphs. (4 marks)

   c) Why is Tarjan's Algorithm a depth-first Algorithm for finding strongly connected components? Your answer should explain the significance of the 'LowLink' value calculated as a part of the algorithm.

   (6 marks)

   d) Show that it is possible to calculate LowLink in constant time. (4 marks)

   e) Explain why the output from Tarjan's Algorithm is topologically sorted.
(4 marks)

2. a) Explain how to solve two-dimensional linear programming problems graphically. You should explain any restrictions on the system you are solving. (2 marks)

   b) Give a generalized description of the problem in matrix form for higher dimensional problems.

   (2 marks)

   c) Give an algorithm for the Simplex Method. (4 marks)

   d) Apply the Simplex Method to the following problem.

$$
\begin{array}{lllll}
\text{Maximize} & f = 3x + 4y + 2z + v & & \\
\text{subject to} & 3y + 4z & <= & 10 \\
 & x + z & <= & 3 \\
 & 3x + 5y & <= & 4 \\
 & 10x + v & <= & 10 \\
 & x + y + z + v & <= & 4 \\
\text{and} & x \geq 0,\ y \geq 0,\ z \geq 0,\ v \geq 0 & &
\end{array}
$$

   (6 marks)

   e) How can the algorithm be adjusted if some of the parameters are restricted to be integers? (2 marks)

   f) What is the optimal solution if all of the variables are restricted to be integers?

   (4 marks)

Section B

Use separate answer books for the two sections.

3. Deterministic and non-deterministic complexity classes

   a) Define the following complexity classes

      i) PTIME;
      ii) NPTIME;
      iii) PSPACE;
      iv) NPSPACE.

   (4 marks)

   b) Give examples of logics for which the satisfiability problem is known to be

      i) PTIME-complete
      ii) NPTIME-complete
      iii) PSPACE-complete.

   (If you are not sure of the standard name of the logic you have in mind, you may simply define it without loss of marks.) (3 marks)

   c) If $\mathcal{C}$ is a class of problems over some alphabet Σ, define what is meant by CO-$\mathcal{C}$. (2 marks)

   d) Explain informally why the following equations hold.

   $$\begin{aligned} \text{PTIME} &= \text{CO-PTIME} \\ \text{PSPACE} &= \text{CO-PSPACE} \end{aligned}$$

   You should make it clear why this explanation does *not* show the equality of NPTIME and CO-NPTIME. (5 marks)

   e) Savitch's Theorem states the following: If $f : \mathbb{N} \to \mathbb{N}$ is a function such that $f(n) \geq \log_2 n$ for all n, then $\text{NSPACE}(f) \subseteq \text{SPACE}(f^2)$. Explain carefully why this shows

   $$\text{PSPACE} = \text{NPSPACE}.$$

   (4 marks)

   f) Using this fact, show that

   $$\text{NPSPACE} = \text{CO-NPSPACE}.$$

   (2 marks)

4. Graph-theoretic problems

   a) Let $G = (V, E)$ be a finite connected (undirected) graph. Explain what is meant by saying that G has an *Eulerian circuit*. (3 marks)

   b) Explain what is meant by saying that G has a *Hamiltonian circuit*. (3 marks)

   c) State a simple necessary and sufficient condition for G to have an Eulerian circuit. (You need not prove that this condition is necessary and sufficent.) Explain why it follows that the problem

      <u>EULER-CIRC</u>:

      > Given: A graph, G
      > Return: Y, if G has an Eulerian circuit; N, otherwise.

      is in PTIME. (2 marks)

   d) What is the *Travelling salesman problem* (TSP)? (2 marks)

   e) It is often said that the TSP is NPTIME-complete. Explain *exactly* in what sense this is so. Your answer should take account of the fact that, strictly speaking, the notion of NPTIME-completeness applies to decision-problems—i.e. problems with YES/NO answers. Why is this a fair relaxation of this notion? (4 marks)

   f) Using the fact that the problem

      <u>HAMILTON-CIRC</u>:

      > Given: A graph, G
      > Return: Y, if G has a Hamiltonian circuit; N, otherwise.

      is NPTIME-complete, show that TSP is also NPTIME-complete, in the sense you have explained. (6 marks)

END OF EXAMINATION