COMP 21111

Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Logic and Modelling

Date: Wednesday 25th January 2017

Time: 09:45 - 11:45

Please answer any THREE Questions from the FOUR Questions provided

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This is a CLOSED book examination

The use of electronic calculators is NOT permitted

[PTO]

1.

a) Consider the following formula:

$$r \wedge \neg p \rightarrow ((p \leftrightarrow \neg q) \rightarrow r).$$

1. Draw the parse tree for this formula. (4 marks)
2. Indicate polarities of all subformula occurrences. (2 marks)
3. Write down the positions of all occurrences of variable p in this formula. (2 marks)
4. Apply the optimised definitional clausal normal form transformation to this formula. (6 marks)

b) Explain briefly how validity and equivalence of propositional formulas can be expressed in terms of satisfiability. (2 marks)

c) Consider n different propositional variables $p_1, \ldots, p_n$, where $n \geq 2$. What is the number of different models of the formula below:

$$p_1 \rightarrow (p_2 \rightarrow (p_3 \rightarrow \ldots \rightarrow p_n) \ldots)$$

Explain your answer. (4 marks)

2.

a)

1. What is a pure literal in a set of clauses? (1 marks)

2. Check satisfiability of the following set of clauses using only the pure literal rule. If this set is satisfiable write down a model.

$$\begin{array}{l} \neg p \lor q \lor \neg s \\ p \lor \neg r \lor s \\ p \lor \neg q \lor \neg s \\ \neg p \lor \neg q \\ \neg r \lor \neg q \end{array}$$

(3 marks)

b) Consider the set consisting of the following clauses:

$$p_1 \lor \neg p_2 \lor \neg p_3, \quad \neg p_1 \lor p_3, \quad p_1 \lor p_2, \quad p_2 \lor p_3, \quad p_1 \lor \neg p_2 \lor p_3.$$

1. Show how the WSAT algorithm can find a model of this set starting with the initial random interpretation $\{p_1 \mapsto 0, p_2 \mapsto 0, p_3 \mapsto 0\}$. (6 marks)

2. For each variable show the probability of flipping this variable at the first step of the WSAT algorithm. (2 marks)

c) Check the validity of the following formula using semantic tableaux:

$$(p \lor r) \to ((p \to q) \to r \lor q).$$

(8 marks)

3.

a)

Draw the OBDD for the formula $(\neg p_1 \rightarrow p_3) \leftrightarrow (p_1 \vee \neg p_2)$ and the order $p_3 > p_2 > p_1$. (6 marks)

b)

1. Write down the definition of a Horn clause. (1 marks)
2. Consider a set of Horn clauses S such that each clause in S contains at least two literals. Show that S is satisfiable. (2 marks)

c) Consider the following QBF formula in CNF:

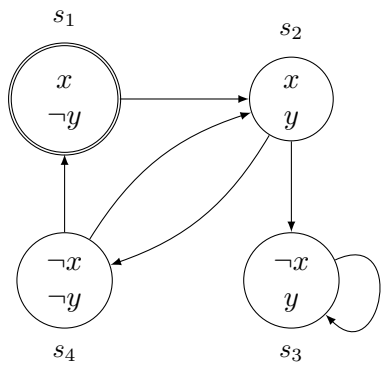
$$\exists s \forall p \exists r((\neg s \vee \neg p \vee r) \wedge (\neg s \vee p \vee \neg r) \wedge (\neg s \vee \neg p \vee \neg r) \wedge \\ (s \vee \neg p \vee r) \wedge (s \vee \neg p \vee \neg r))$$

Evaluate this formula using the DPLL algorithm. Show all steps of the algorithm. Is this formula true or false? (8 marks)

d) Let propositional formulas $I(\bar{x})$ and $T(\bar{x}, \bar{x}')$ symbolically represent an initial condition and a transition relation, respectively. Write down a formula representing states reachable in exactly 1 step from the initial states. (3 marks)

4.

a) Consider a transition system with the following state transition graph:

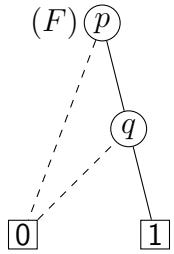


Which of the following formulas are true on at least one path starting from the initial state? If a formula is true on some paths draw one such path.

1. $\Box(x \vee \neg y)$
2. $\Box x \vee \Box \neg y$
3. $\Diamond \Box \neg y$
4. $\Box(x \rightarrow \bigcirc \Diamond y)$
5. $\Diamond x \leftrightarrow \Box y$
6. $x \,\mathbf{U}\, (\Box(x \leftrightarrow y))$

(6 marks)

b) A formula F has the OBDD shown below. Apply the quantification algorithm to this OBDD to obtain OBDD nodes for the formulas $\exists q F$ and $\exists p F$. Integrate obtained OBDD nodes into the DAG for F. (6 marks)



c) A variable x in propositional logic of finite domains has the domain $\{a, b, c\}$. Translate the following PLFD formula into a formula in propositional logic:

$$(\neg x = a \leftrightarrow x = b) \rightarrow x = c.$$

(5 marks)

d) Briefly explain the main difference between GSAT and WSAT algorithms. (3 marks)

END OF EXAMINATION