COMP 21111

Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Logic and Modelling

Date: Thursday 25th January 2018

Time: 09:45 - 11:45

Please answer all THREE Questions.

This is a CLOSED book examination

The use of electronic calculators is NOT permitted

[PTO]

1.

a) Apply the DPLL method to the following set of clauses, i.e., show the DPLL tree and the results of all unit propagation steps. Is this set of clauses satisfiable? If yes, give an interpretation which satisfies it. (9 marks)

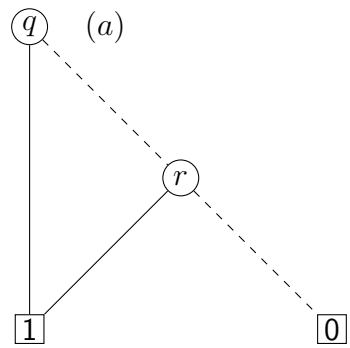
$$\boxed{\begin{array}{r} p \lor \neg q \lor \neg r \\ \neg p \lor q \lor r \\ p \lor q \lor r \\ \neg p \lor \neg q \\ q \lor \neg p \lor \neg r \\ p \lor q \lor \neg r \end{array}}$$

b) Convert the formula $p \land q \leftrightarrow \neg p \lor \neg q$ into clausal normal form using the definitional clausal form transformation. (6 marks)

c) Present the Walk SAT (WSAT) algorithm for propositional satisfiability as a pseudo-code. (5 marks)

2.

a) Consider the following global OBDD dag where $q > r$.



i. Integrate OBDD for a formula $\neg r \to (\neg q \to r)$ into this dag. Denote by (b) the node corresponding to this formula in the new dag. (4 marks)

ii. Apply OBDD conjunction and disjunction algorithms to nodes (a) and (b). Denote resulting nodes in the dag as $(a) \wedge (b)$ and $(a) \vee (b)$ respectively. (4 marks)

b) Transform the following QBF formula into prenex normal form. (5 marks)

$$\exists q(\forall p(p \to q) \to \forall p(\neg q \to p)).$$

c) Apply the splitting algorithm to evaluate the following QBF formula:

$$\forall s \exists q \forall p(q \wedge p \vee (\neg p \leftrightarrow s)).$$

Is this formula true or false? (7 marks)

3.

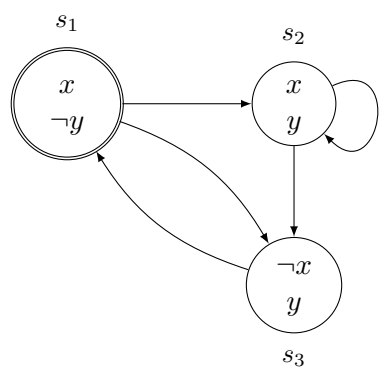
a) A variable x in propositional logic of finite domains has the domain $\{b, e, d\}$. Using the tableau method, check whether the formula

$$\neg(\neg \mathsf{x} \in \{b\} \rightarrow \neg \mathsf{x} \in \{d, e\} \wedge \mathsf{x} \in \{e, b\})$$

is satisfiable. If this formula is satisfiable give a model for it.

(9 marks)

b) Consider a transition system with the following state transition graph:



Which of the following formulas are true on at least one path starting from the initial state? If a formula is true on some paths draw one such path.

1. $\Diamond x \wedge \Diamond \neg x$
2. $\Box(x \rightarrow \neg y)$
3. $\Box(x \rightarrow \Diamond \neg y)$
4. $\Diamond(\neg x \wedge \bigcirc \Box x)$
5. $\Box \Diamond(x \leftrightarrow \neg y)$

(5 marks)

c) Consider a transition system with the following state transition graph as in question 3.b). Write down symbolic representation of:

1. the set of initial states;
2. the set of states reachable in exactly two steps;
3. the transition relation: $\{(s_1, s_2), (s_2, s_2)\}$.

(6 marks)