COMP 26120

One and a half hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Algorithms and Imperative Programming

Date: Monday 15th January 2018

Time: 14:00 - 15:30

Please answer BOTH Questions.

Use a SEPARATE answerbook for each QUESTION

This is a CLOSED book examination

The use of electronic calculators is permitted provided they are not programmable and do not store text

[PTO]

1. Algorithm design.

   For each of the computational tasks (a), (b) and (c) below,

   (i) describe an algorithm for the task. Your description may be a program in a standard language, in pseudocode, or a clear and precise step-by-step description. You should explain your algorithm and why it works. Marks are awarded for a correct algorithm. Some marks are also awarded for efficiency: the more efficient your algorithm is, the more marks it will be awarded.

   (ii) give the worst-case time complexity of your algorithm in terms of the size of the input and the number of operations required. Use the O-notation to express the complexity and explain how you calculated your answer.

   a) A specialist sorting algorithm

   Input: A list s, of length N, of integers *already sorted into ascending order*, and k additional integers, where k is much smaller than N.

   Output: The list in ascending order of the combination of all elements of s together with the k additional integers.

   No marks will be awarded for using a standard sorting algorithm on the list of all elements. You need to devise an efficient algorithm for this case. (5 marks)

   b) Remove duplicate elements from a list of integers

   Input A list of integers.

   Output: A *list of the same integers*, but where each integer occurs only once in the output (the order of elements in the output list is not important). (7 marks)

   c) Interchanging parts of an array

   Input: An array of characters and a division point between two characters in the array, dividing the array into two parts.

   Output: An array consisting of the two parts of the original array swapped (the first put second, and the second put first), but maintaining the order of characters in each of the parts.

   Thus if the array of characters is *abcdefg* and the division point is between c and d, then the interchange results in the array of characters *defgabc*. Extra marks will be awarded if you can do this interchange **in-place**, i.e. by swapping pairs of elements in the array. [Hint: One approach to an in-place algorithm is to consider how the operation of reversing an array may contribute to the interchange of parts of the array. A different approach to an in-place algorithm is to observe that interchanging parts of equal length can be done easily by swapping pairs of elements, and then using this operation repeatedly on smaller parts, if necessary.] (8 marks)

2. Complexity, determinants and permanents

   a) Let $f : \mathbb{N} \to \mathbb{N}$ be a function, where $\mathbb{N} = \{0, 1, \dots\}$. Give explicit definitions of (i) $O(f)$, and (ii) $\Omega(f)$. (2 marks)

   b) Show formally that $f \in O(g)$ if and only if $g \in \Omega(f)$. (2 marks)

   c) Give a recursive definition of the *determinant* of an $n \times n$ matrix

$$A = \begin{pmatrix} a_{1,1} & \dots & a_{1,n} \\ \vdots & & \vdots \\ a_{n,1} & \dots & a_{n,n} \end{pmatrix}$$

   in terms of the determinants of smaller matrices. (4 marks)

   d) Given an alternative definition of the determinant of A in terms of the entries $a_{i,\sigma(i)}$, where $1 \leq i \leq n$ and σ is a permutation of $\{1, \dots, n\}$. (2 marks)

   e) Explain how to compute the determinant of A in time $O(n^3)$ using triangulation. You should explain why your algorithm gives the correct result and show that it has the stated complexity bound. You may use any standard facts relating to determinants without proof, as long as they are clearly stated.
   (7 marks)

   f) What is meant by the *permanent* of an $n \times n$ matrix A? What is known about the time taken to calculate this quantity? (You may wish to use Ω-notation.) You need not justify your answer. (3 marks)

END OF EXAMINATION