Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Logic and Modelling

Date: Tuesday 22nd January 2019

Time: 09:45 - 11:45

Please answer all THREE Questions.

This is a CLOSED book examination

The use of electronic calculators is NOT permitted

1.

a) Consider the following formula:

$$(r \leftrightarrow \neg q) \rightarrow (r \wedge \neg q \rightarrow p).$$

1. Draw the parse tree for this formula. (4 marks)
2. Indicate polarities of all subformula occurrences. (2 marks)
3. Write down the positions of all occurrences of variable q in this formula. (2 marks)
4. Apply the optimised definitional clausal normal form transformation to this formula. (6 marks)

b) Describe briefly two main differences between the splitting and the DPLL algorithms for propositional satisfiability. (2 marks)

c) Formalise the following problem in propositional logic. Given n jobs and k workers is it possible to assign jobs to workers such that (i) each job is assigned to at most one worker, (ii) no two workers have the same job and (iii) every job is assigned to some worker. (4 marks)

2.

a) Consider the Walk SAT algorithm (WSAT).

i Explain briefly how the WSAT algorithm selects a variable for flipping.

(4 marks)

ii Consider the following set of clauses:

$$\begin{gathered} \neg p_0 \vee p_1 \vee p_2 \\ \neg p_0 \vee p_1 \vee \neg p_3 \\ \neg p_0 \vee \neg p_1 \vee p_3 \\ \neg p_0 \vee p_2 \end{gathered}$$

Compute the probabilities of flipping for each of the variables p_0, p_1, p_2, p_3 at the first step of the WSAT algorithm applied to this set of clauses and the initial interpretation

$\{p_0 \mapsto 1, p_1 \mapsto 0, p_2 \mapsto 0, p_3 \mapsto 1\}$.

(4 marks)

b) Describe briefly how to check validity of a propositional formula F using the semantic tableaux method. Use 2-4 sentences. (2 marks)

c) Consider the following QBF formula in CNF:

$$\boxed{\begin{gathered} \exists p \forall q \exists r \\ \neg p \vee q \\ \neg p \vee \neg q \vee \neg r \\ \neg p \vee q \vee \neg r \\ p \vee q \vee r \\ p \vee q \vee \neg r \end{gathered}}$$

Evaluate this formula using the DPLL algorithm. Show all steps of the algorithm. Is this formula true or false? (10 marks)

3.

a) Consider the following formula:

$$(q \to \neg p) \to (\neg p \wedge \neg q)$$

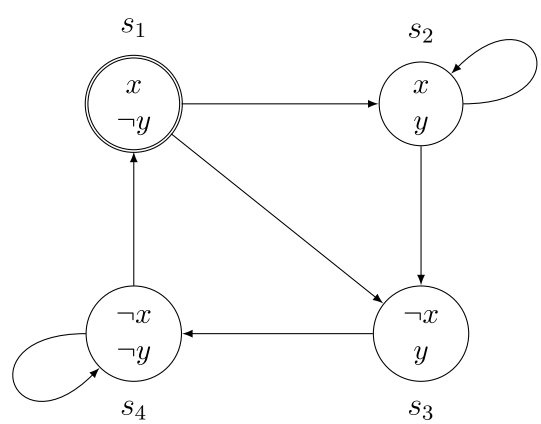
i Draw the OBDD for this formula with the ordering $p > q$. (5 marks)

ii Based on the constructed OBDD, eliminate the existential quantifier from the formula:

$$\exists q((q \to \neg p) \to (\neg p \wedge \neg q)).$$

(3 marks)

b) Consider a transition system with the following state transition graph.



Which of the following formulas are true on at least one of the paths starting from the initial state? If a formula is true on a path, draw one such path.

i $\Box(\neg y \vee \Box y)$ (1 marks)

ii $\Diamond(y \wedge \bigcirc x \wedge \bigcirc\bigcirc \Box \neg x)$ (1 marks)

iii $\Box\Diamond(x \leftrightarrow \neg y)$ (1 marks)

iv $\Diamond\Box(x \leftrightarrow \neg y)$ (1 marks)

v $(x \leftrightarrow \neg y)\, \mathbf{U}\, (\Box \neg y)$ (1 marks)

vi $\Box(\neg y\, \mathbf{U}\, x)$ (1 marks)

c) Consider formulas $I(\bar{x})$ and $T(\bar{x}, \bar{x}')$ symbolically representing a set of initial states and a transition relation.

i Define the formula $R_{=1}(\bar{x})$ representing the set of states reachable from the initial states in exactly one step. (2 marks)

ii Define formulas $R_{\leq 0}(\bar{x}), \ldots, R_{\leq n}(\bar{x})$ representing sets of states reachable from the initial states in $\leq 0, \ldots, \leq n$ steps, respectively. (4 marks)