Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Advanced Algorithms 1

Date: Friday 18th January 2019
Time: 14:00 - 16:00

Please answer all THREE Questions.

This is a CLOSED book examination
The use of electronic calculators is NOT permitted
1. a) Define the \textit{string matching problem.}  

b) Describe the \textit{Rabin-Karp algorithm} for the string matching problem. For full marks give an explicit algorithm. 

c) Discuss the complexity of the Rabin-Karp algorithm. 

d) Define the term \textit{primitive recursive function.}  

e) Give a definition of the \textit{Ackermann function.} (Any one of many definitions of functions known as the Ackermann function will do.) 

f) Write down the union-find algorithm using trees and path compression. 

g) What is the relationship between the complexity of the algorithm in part f) and the Ackermann function?
2. a) Define the terms flow network and flow for a flow network and auxiliary graph for a flow (for a flow network). (6 marks)

b) The following graph shows a flow (in parentheses) for a flow network with capacities outside the parentheses.

Draw the auxiliary graph of the given flow. Derive a maximum flow and draw (in a separate diagram) its flow graph. Draw (in a separate diagram) the auxiliary graph of your maximum flow. (5 marks)

c) What property does the auxiliary graph of your maximum flow have? (1 mark)

d) Give careful definitions of the problems: PROPOSITIONAL SAT, SAT, k-SAT. (4 marks)

e) Write down the DPLL algorithm for one of the problems in your answer for part d). (2 marks)

f) Why is the SAT problem important? (2 marks)
3. a) If $\mathbf{P}$ is a class of problems, what is meant by $\text{Co-}\mathbf{P}$ (over an alphabet $\Sigma$)?
   
   (2 marks)

b) Explain the terms (i) NPTIME, (ii) NPTIME-hard, (iii) NPTIME-complete.
   
   (6 marks)

c) Give a many-one polynomial time reduction from SAT to 3-SAT. Explain the correctness of your solution.
   
   (4 marks)

d) Define the problem INTEGER LINEAR PROGRAMMING FEASIBILITY (ILP).
   
   (4 marks)

e) Give a many-one polynomial time reduction from SAT to ILP. Explain the correctness of your reduction, and deduce a lower complexity bound for ILP.
   
   (4 marks)

END OF EXAMINATION