Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Quantum Computing

Date: Thursday 4th June 2009
Time: 09:45 – 11:45

Please answer any THREE Questions from the FIVE questions provided

This is a CLOSED book examination

The use of electronic calculators is NOT permitted.
1. a) Let $|x\rangle$ and $|y\rangle$ be two vectors in $\mathbb{C}^n$. How is their inner product defined? (3 marks)

b) What does the phrase “$|x\rangle$ and $|y\rangle$ are orthogonal” mean? (2 marks)

c) Write down the three Pauli matrices $\sigma_x$, $\sigma_y$, $\sigma_z$. (3 marks)

d) What are the eigenvalues and eigenvectors of $\sigma_z$, $\sigma_x$? (4 marks)

e) What does it mean for an operator to be: hermitian, unitary? (2 marks)

f) Write down the Hadamard matrix $H$. (2 marks)

g) Prove that $H^{\otimes n} |w\rangle = \left(\frac{1}{\sqrt{2}}\right)^n \sum_z (-1)^{w.z} |z\rangle$. (4 marks)

2. a) What is the state of a quantum system? (2 marks)

b) Suppose $A = |2\rangle \langle 2| + |3\rangle \langle 3| + |4\rangle \langle 4|$ where we have written down the eigenvector expansion of the hermitian operator $A$. Suppose $|\psi\rangle = \frac{1}{2} |2\rangle + \frac{\sqrt{3}}{2} |4\rangle$.

What happens when $A$ is measured in the state $|\psi\rangle$, and what is the expectation value of $A$ in $|\psi\rangle$? (4 marks)

c) Now suppose

$A = |2\rangle \langle 2| + |3a\rangle \langle 3a| + |3b\rangle \langle 3b|$, and that $|\psi\rangle = \frac{1}{2} |2\rangle + \frac{1}{2} |3a\rangle + \frac{1}{\sqrt{2}} |3b\rangle$.

What happens when $A$ is measured in the state $|\psi\rangle$, and what is the expectation value of $A$ in $|\psi\rangle$? (4 marks)

d) State and prove the No-Cloning Theorem. (2 marks)

e) If $n$ is a unit vector in real 3-D space, write down the matrix $n \cdot \sigma$. (3 marks)

f) Write down the expectation values of $n \cdot \sigma$ in the states $|+z\rangle$, $|-z\rangle$, $|+\chi\rangle$, $|-\chi\rangle$ (which refer to the +1 and −1 eigenvectors of $\sigma_z$ and $\sigma_x$ respectively. (5 marks)
3. a) Is the state \( |\psi_A\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |01\rangle) \) separable or entangled? Explain your answer. (2 marks)

b) Is the state \( |\psi_B\rangle = \frac{1}{\sqrt{2}} (|10\rangle + |01\rangle) \) separable or entangled? Explain your answer. (2 marks)

c) Write down the vectors that constitute the Bell Basis. (4 marks)

d) Let \( A, B, C, D \) be four systems, each capable of yielding +1 or -1 when measured. Assuming a local realistic world, and that \( A, B, C, D \) are widely separated, derive the CHSH inequality. (8 marks)

e) Explain how quantum mechanics violates the CHSH inequality. (4 marks)

4. a) How do the four standard basis vectors get mapped by the CNOT gate? (2 marks)

b) Write down the matrix of the CNOT gate. (2 marks)

c) Describe what happens to an arbitrary vector in \( \mathbb{C}^2 \otimes \mathbb{C}^2 \) when the first qubit is measured. (2 marks)

d) Describe the Toffoli gate. How is it used to implement a NAND gate? (4 marks)

e) Give a clear account of the Deutch-Jozsa algorithm. (10 marks)

5. Give a detailed account of the quantum Fourier transform. (20 marks)

END OF EXAMINATION