Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Mathematics for Computer Systems Engineers

Date: Friday 28th May 2010
Time: 14.00 – 16.00

Please answer THREE questions from the FOUR provided
Use separate Answerbooks for EACH section
This is a CLOSED book examination

The use of electronic calculators is NOT permitted.
Section A

1. a) Let $A$ be the propositional logic formula

\[(p \Rightarrow (q \land r)) \land ((p \land q) \Rightarrow r)\]

For parts (i) and (ii) simplify your answers where possible.

i) Find a formula in CNF form that is logically equivalent to $A$.  
   (6 marks)

ii) Find a formula in DNF form that is logically equivalent to $A$.  
    (4 marks)

iii) Is $A$ a tautology? Is $A$ satisfiable?  
     (2 marks)

b) Let the predicates $M$, $T$ be defined by

$T(x, y)$ means $x$ has taken the course unit $y$
$P(x, y)$ means $x$ has passed the course unit $y$
$C(x)$ means $x$ is a CSE student

Express each of the following using quantifiers

i) No CSE student has taken COMP10042.  
   (8 marks)

ii) All CSE students have passed COMP10031.  
    (8 marks)

iii) There are two CSE students who have passed at least one unit.  
    (8 marks)

iv) All CSE students have passed all the units they have taken.  
    (8 marks)
2. a) Prove by induction that, for all integers $n \geq 0$

\[ \sum_{i=0}^{n} i2^i = (n-1)2^{n+1} + 2 \quad \text{(8 marks)} \]

b) Let $p(n)$ be the statement “$n^2 - 7n + 12 \geq 0$”. Use induction to prove that $p(n)$ is true for all for integers greater than 2. \( \text{(6 marks)} \)

c) Define functions from $\mathbb{N}$ to $\mathbb{N}$ with each of the properties

i) Injective but not surjective.

ii) Surjective but not injective.

iii) Bijective but not the identity function.

In each case explain why the functions have the desired properties. \( \text{(6 marks)} \)
Section B

3. a) Find the determinant of the matrix $A$ below.

$$
A = \begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 \\
1 & 2 & 3 & 4 \\
5 & 1 & 0 & 2
\end{bmatrix}.
$$

(6 marks)

b) Let $a, b, c$ be real numbers and let

$$
B = \begin{bmatrix}
a & 0 & 0 \\
0 & b & 1 \\
2 & 0 & c
\end{bmatrix}.
$$

Suppose that $\det(B) = 7$. Find $\det(C)$ for the matrix

$$
C = \begin{bmatrix}
2 & 0 & c \\
4a & 0 & 0 \\
0 & 3b & 3
\end{bmatrix}.
$$

(8 marks)

c) For the matrix

$$
C = \begin{bmatrix}
1 & 2 & 0 & 0 \\
0 & 2 & 1 & 1 \\
0 & 0 & 3 & 0 \\
0 & 0 & 0 & 4
\end{bmatrix},
$$

Find $C^{-1}$. (6 marks)
4. a) Find the MacLaurin series expansion for \( f(x) = e^{2x} \). What is the radius of convergence? (10 marks)

b) Find

\[
\lim_{n \to \infty} \left( \frac{\sin\frac{1}{n}}{\ln\frac{n}{n}} \right)
\]

(10 marks)