COMP34120

Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

AI and Games

Date: Friday 20th May 2011

Time: 09:45 - 11:45

Please answer any THREE questions from the FOUR questions provided

Use a SEPARATE answerbook for each Question

Marks will be awarded for reasoning and method as well as being correct.

This is a CLOSED book examination

The use of electronic calculators is NOT permitted

1. a) Find all the pure equilibrium points for the following game, where the pay-off function for Player i, P_i is given for each choice of strategy (s_1, s_2, s_3) by all participants. How do you think the game will be played in practice? (4 marks)

	P_1	P_2	P_3
$(0,0,0)$	1	0	-1
$(0,0,1)$	-1	1	0
$(0,1,0)$	0	-1	1
$(0,1,1)$	1	1	-1
$(1,0,0)$	-1	-1	1
$(1,0,1)$	-1	0	1
$(1,1,0)$	0	1	-1
$(1,1,1)$	-1	0	0

b) Consider the game played by the Congress and the Federal Reserve. The pay-offs are given in approval ratings on a scale of 1 to 4 in the following matrix.

		Federal Reserve	
		low interest rates	high interest rates
Congress	budget balanced	(3,4)	(1,3)
	budget deficit	(4,1)	(2,2)

i) Remove dominated strategies from the game one by one. What size matrix do you have at the end? (3 marks)

ii) What can you say about the choices of strategy left? Do you think these strategies are good choices for both parties? (2 marks)

c) In a 2-person zero-sum game with at least 2 different pure strategy equilibrium points what can you say about the number of mixed strategy equilibrium points? (4 marks)

d) Given a 2-person zero-sum game in matrix form and a choice of mixed strategy for each of the players, how would you check that combination is an equilibrium point? (3 marks)

e) You are charged with finding a good strategy for a 2-person game. Under which circumstances would it be advantageous to have the game in normal form, and when would it be preferable to have it in extensive form? For full marks your answer should include justifications. (4 marks)

2. Pick a reasonably well-known board game of your choice, excluding Kalah.

   a) What can you say about the type of the game you have chosen? (2 marks)

   b) What would constitute a solution to your chosen game? What properties would you expect such a solution to have? (4 marks)

   c) How would you go about developing a program that is a strong player of your chosen game? Try to give as much detail as possible. (14 marks)

3. a) What is the definition of a two-person Stackelberg game and Stackelberg strategy? (2 marks)

b) Find the Stackelberg strategy for the following Stackelberg game:

- There are two players in which Player L is the leader and Player F is the follower;
- The strategy spaces for the leader and the follower are $U_L = [1, +\infty)$ and $U_F = [1, +\infty)$;
- The payoff functions for the leader and the follower are

$$J_L(u_L, u_F) = (u_L - 1)(5 - 3u_L + u_F)$$
$$J_F(u_L, u_F) = (u_F - 1)(1 + 2u_L - u_F)$$

in which $u_L \in U_L$ is the leader's strategy and $u_L \in U_L$ is the follower's strategy.

(Note. 7 marks are for the step by step process to obtain the solution, 2 marks for the correct answer.) (9 marks)

c) Consider the two person games in which, firstly, Player 1 has the choice to play either a Nash game or a Stackelberg game; secondly, the pure-strategy for Nash equilibrium exists and is unique; thirdly, Stackelberg strategy exists. Then answer the following questions:

c.1) What does Player 1 have to do in order to become the leader? (1 mark)

c.2) Which of the following statements is true?

A. Player 1 is always better off if he plays Nash games

B. Player 1 is always better off if he plays Stackelberg games as the leader

C. Player 1 is always better off if he plays Stackelber games as the follower

D. No matter what Player 1 chooses, he can only be better off in some games but worse off in others

where the "better off" means that that Player 1's payoff for playing one game is larger than or at least equal to the payoff for playing another game. (2 marks)

c.3) Justify your answer to question c.2) above. If your answer is D, then you need to give examples to support your answer; If your answer is A, B, or C, then you need to verify or prove your answer. (6 marks)

4. A *lowest unique-bid* auction works as follows. Each participant submits a sealed (secret) bid. The bid which is *lowest* and unmatched by any other other bid wins the item. In other words, for a bid to win, it must be the case that only one person bid that precise amount, and for all lower bids more than one person bid that precise amount. In such auctions, the winner often wins the opportunity to buy an item at a price much below its market value.

   We want you to consider the following version of a lowest unique-bid auction. The value of the item is W. The auction ends when exactly n bids have been submitted. All bids must be in exact pounds, e.g. £1, £2, ... The winner pays the bid and gets the item; all other participants pay £1 and get nothing. If there is no winner (because no bid is unmatched), nobody pays anything and nobody gets anything.

   a) What is the equilibrium when the number of participants n is limited to 2? (HINT: write out the game in normal form, just considering the three actions, bid £1, £2, £3 to see the pattern.) (4 marks)

      Will this solution work when the number of players $n = 3$? (2 marks)

   b) Now consider the auction when $W = 50$ and $n = 100$. Describe how an evolutionary algorithm could be used to look for equilibria of this game. For full marks give algorithmic detail, otherwise at least name the approach you would use. Is this approach guaranteed to find an equilibrium? (7 marks)

   c) Suppose you participate in such an auction, again $W = 50$ and $n = 100$, among a community who runs these auctions repeatedly in order to raise money for charity. Describe how a value-function estimate reinforcement learning algorithm can be used to find a good bidding strategy. For full marks give algorithmic detail, otherwise at least name the approach you would use. Is this approach guaranteed to find an optimal solution? (7 marks)

END OF EXAMINATION