Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Advanced Algorithms II

Date: Wednesday 1st June 2011
Time: 14:00 - 16:00

Please answer ONE Question from EACH Section
(a total of THREE questions from the SIX Questions provided)

Use a SEPARATE answerbook for each SECTION

For full marks your answers should be concise as well as accurate.
Marks will be awarded for reasoning and method as well as being correct.

This is a CLOSED book examination

The use of electronic calculators is permitted provided
they are not programmable and do not store text

[PTO]
You should answer all the subquestions in the selected question

1. Question 1 (25 marks)
   Complex network models and their topological properties. Explain the basic models of complex networks. You should include in your answer (at least) the following information:

   a) The basic models of network topologies, including the basic algorithms for obtaining each of them. How do the input parameters of the algorithms influence the type of network obtained? Are there any extensions of the basic algorithms that you have described? (12 marks)

   b) Basic topological properties of each of the network models explained above. You will mention, at least, the basic topological properties related to the size, density and connectivity of the network. Compare the basic network models with each other. (8 marks)

   c) Examples of real-world complex networks with the properties and topologies you have previously described. (5 marks)
2. Question 2 (25 marks)

Synchronisation of complex dynamical networks. Explain the basic models for dynamical synchronisation in complex networks. You should include in your answer (at least) the following information:

a) What is synchronisation in complex dynamical networks? What are the main elements to consider in order to establish synchronisation in a complex dynamical network? What is a dynamical network in this context? (6 marks)

b) Types of synchronisation and models to describe it. (6 marks)

c) Describe the main models of synchronisation of coupled-dynamical oscillators in networks. (8 marks)

d) Does the network topology influence the synchronisability of networks? If so, how? (5 marks)
Section B

Answer one of the two questions

You should answer all the subquestions in the selected question

1. Question 3 (25 marks)

There are many different optimisation algorithms, each one following a specialised strategy to localize optima of functions. While it is common to discuss their details, it is also important to recognise that they are fundamentally similar. This type of abstraction helps identify parts that are common to all algorithms which will make implementations more efficient. Considering only the algorithms that work on a single solution (i.e. ignoring population-based algorithms), describe the aspects of these algorithms that make them similar to each other. You should cover at least the following issues:

a) the types of problems for which optimisation algorithms are useful, including how the problem is represented in the algorithm.

b) You should describe the main iteration cycle that these algorithms follow.

c) derivative-based algorithms for multi-dimensional problems rely on solving optimisation problems in one dimension. Describe which approaches exist to find optima in a single dimension and provide a detailed description of one of these algorithms (include enough detail to allow a programmer to create an implementation from your description; you can use pseudo-code if you like, but a textual description is also appropriate).
2. Question 4 (25 marks)

Population-based algorithms are quite popular particularly for global optimisation problems. Perhaps the most studied and widely used of these are the evolutionary algorithms. Describe the generic approach of evolutionary algorithms, starting with the general concepts and then provide details of the major components. You should at least focus on the following points:

a) Describe the main characteristics of evolutionary algorithms that make them efficient at escaping local minima, including the dicotomy of diversity versus selection. (6 marks)

b) Describe the general iterative approach of all evolutionary algorithms. (6 marks)

c) Describe the main operators used by evolutionary algorithms. (5 marks)

d) Discuss how genetic programming is different from the other evolutionary algorithms and provide a description of how the mutation and recombination operators could be implemented. (8 marks)
3. Question 5 (25 marks)

a) Give a representation of a floating point number in terms of 4 integers (the mantissa, the base, the precision, and the exponent). Explain the difference between a normalised and a denormalised number. (2 marks)

b) Represent the decimal number $x = 0.1$ as a single precision floating point number $\bar{x}$. Use $b = 2$ and a simple truncation. What is the absolute error $e = |\bar{x} - x|$ in this case? (4 marks)

c) Write an expression for the bound of the absolute error for the summation $y = x_1 + x_2 + x_3 + x_4$. Use graphs to present the error propagation through the computation. Assume that $0 < x_1 < x_2 < x_3 < x_4$ and that the numbers $x_i$, $i = 1, 2, 3, 4$ are given exactly (i.e. $\epsilon_{x_i} = 0$, $i = 1, 2, 3, 4$, so that the only errors come from the summation). Based on the obtained expression for $|\epsilon'_y|$ derive the way of performing the summation that minimises the bound of the relative error. (10 marks)

d) Write the expression that gives an estimate of the number of significant digits when a number $x$ is approximated by $\bar{x}$. Write an expression for the estimate of the number of significant digits $\ell_z$ in the result $z = f(x_1, \ldots, x_m)$, where $f : R^m \rightarrow R$ is an arbitrary function. Assume that the operands $x_i$ are approximated by $\bar{x}_i$ with $\ell_i$ significant digits and that $\ell_{\min} = \min(\ell_1, \ldots, \ell_m)$. (4 marks)

e) Let $z = f(x_1, x_2, x_3, x_4) = x_1 + x_2 + x_3 + x_4$ with

$$x_1 = 0.1001 \cdot 10^1, x_2 = -0.1000 \cdot 10^1, x_3 = 0.1002 \cdot 10^1, x_4 = -0.1001 \cdot 10^1$$

(hence $\ell_i = 4$, $i = 1, 2, 3, 4$). Apply the formulas from Part d) to estimate the loss of significant digits $\delta$ in the result. Suggest a method of summation that avoids the problem. (5 marks)
Consider the following initial value problem:

\[ y' = f(x, y), \quad 0 < x < 2, \quad y(0) = y_0 \]  

(1)

where \( f(x, y) \) is a bounded function on \([0, 2]\).

a) Give a formula for a general linear multistep method of order \( k \) for the numerical solution of the problem (1). What is the difference between explicit and implicit linear multistep methods?  

(4 marks)

d) Give the formulas for determining the order of convergence of a linear multistep method. Give the formulas for the first and the second characteristic polynomials associated with a linear multistep method. Based on these definitions, give the necessary conditions for consistency and stability of a linear multistep method.  

(6 marks)

c) Consider the following linear multistep method:

\[ y_{n+2} - (1 + a)y_{n+1} + ay_n = \frac{h}{2}[(3 - a)f_{n+1} - (1 + a)f_n], \quad n = 0, 1, \ldots \]  

(2)

i) Determine the order of the method (2) as a function of the real parameter \( a \). Is the method consistent?  

(2 marks)

ii) Find the interval of the values of \( a \) for which the method is stable.  

(3 marks)

iii) Consider the application of the following two special cases of the method (2) obtained for \( a = 0 \) and \( a = -5 \) respectively:

\[ a = 0: \quad y_{n+2} - y_{n+1} = \frac{h}{2}[3f_{n+1} - f_n] \quad n = 0, 1, 2, \ldots, \]  

(3)

\[ a = -5: \quad y_{n+2} + 4y_{n+1} - 5y_n = \frac{h}{2}[8f_{n+1} + 4f_n] \quad n = 0, 1, 2, \ldots, \]  

(4)

to the initial value problem:

\[ y' = e^x, \quad 0 < x < 2, \quad y(0) = 1. \]  

(5)

Using \( h = 0.25 \), and taking the following initial values \( y_0 = 1, y_1 = 1.284025 \), report the approximate solutions \( y_2, \ldots, y_8 \) obtained by both methods to 6 decimal places accuracy. Compare the values of the approximate solutions with the exact solution of the problem (5) given by \( y = e^x \). What can be concluded?  

(10 marks)