Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Advanced Algorithms II

Date:    Monday 28th May 2012
Time:    14:00 - 16:00

Please answer ONE Question from EACH Section
(a total of THREE questions from the SIX Questions provided)

Use a SEPARATE answerbook for each SECTION

For full marks your answers should be concise as well as accurate.
Marks will be awarded for reasoning and method as well as being correct.

This is a CLOSED book examination

The use of electronic calculators is permitted provided
they are not programmable and do not store text
1. Question 1 (25 marks)

The minimisation of multivariate functions is often carried out using algorithms based on derivatives.

a) Describe one algorithm of this type, either using a textual description or pseudo-code.

(12 marks)

b) Discuss the similarities between the algorithm you described above and one other of the same class (i.e. also based on derivatives). Compare their advantages and disadvantages.

(13 marks)
2. Question 2 (25 marks)

Genetic programming is an evolutionary algorithm that is used to evolve functions or programs.

a) Describe how the functions or programmes can be represented in genetic programming. Mention the operators that transform them and how the representation must support those operators (13 marks)

b) Describe the main characteristics of evolutionary algorithms, and discuss how the operation of these algorithms is similar to how human programmers work. (12 marks)
Section B

Answer one of the two questions

You should answer all the subquestions in the selected question

1. Question 3 (25 marks)

a) Assume that a real number $x$ is approximated by its floating point representation $\tilde{x}$, with normalised mantissa. If the decimal system is used (base $b = 10$) and the number of digits in the mantissa is $n = 5$, write the expressions for the absolute and relative error in the case when the mantissa is rounded and give a bound for each of these errors. (8 marks)

b) Let the relative errors of representing the real numbers $x$, $y$, and $z$ as floating point numbers $\tilde{x}$, $\tilde{y}$, and $\tilde{z}$ are $r_x$, $r_y$, and $r_z$, respectively. Using error propagation graphs, determine the relative error in computing $w = \frac{x}{y - z}$. In your answer, assume that relative roundoff errors of performing the subtraction and the division are $r_- \cdot r_{\div}$, respectively. (8 marks)

c) Write a formula for the number of significant digits $\ell_w$ in a computation $w = f(x_1, \ldots, x_m)$, where $f : \mathbb{R}^m \rightarrow \mathbb{R}$. In your answer assume that each argument $x_i$, $i = 1, \ldots, m$ has floating point representation $\tilde{x}_i$ with $\ell_i$ significant digits in mantissa, and that $\ell_{\min} = \min(\ell_1, \ldots, \ell_m)$. If $x_1 = 0.14143 \cdot 10^1$ and $x_2 = 0.14142 \cdot 10^1$, estimate the loss of significant digits when computing $w = x_1 - x_2$. (9 marks)
2. Question 4 (25 marks)

Consider the following initial value problem:
\[ y' = f(x, y), \quad 0 \leq x \leq 1, \quad y(0) = y_0, \]  
(1)

where \( f(x, y) \) is a bounded function on \([0, 1]\).

a) Give a formula for a general linear multistep method of order \( k \) for the solution of the problem (1). Give the formulas for determining the order of convergence of a linear multistep method. Finally, give the necessary conditions for consistency and stability of a linear multistep method. (7 marks)

b) Consider the following linear multistep method:
\[ y_{n+4} - \frac{8}{19}(y_{n+3} - y_{n+1}) - y_n = \frac{6h}{19}(f_{n+4} + 4(f_{n+3} + f_{n+1}) + f_n), \quad n = 0, 1, \ldots \]  
(2)

i) Determine the order of convergence of the method (2). (4 marks)

ii) Examine the stability of the method (2). Is the method (2) optimal? (5 marks)

iii) Apply method (2) to the solution of the following initial value problem:
\[ y' = 3x^2 + 4, \quad 0 \leq x \leq 1, \quad y(0) = 1. \]  
(3)

Adopt the step size \( h = 0.2 \) and the following initial values (given with 3 decimal places) \( y_0 = 1.000, \ y_1 = 1.808, \ y_2 = 2.664, \ y_3 = 3.616. \) Compute the approximate solutions \( y_4 \) and \( y_5 \) using (2) with 3 decimal places. Compare the computed values \( y_4 \) and \( y_5 \) with the exact solution of the problem \( y = x^3 + 4x + 1 \) at the points \( x = 0.8 \) and \( x = 1 \), respectively. What can be concluded? (9 marks)
Section C

You should answer all the subquestions in the selected question

1. Question 5 (25 marks)

   a) Imagine the air-traffic network of airports in the USA. This is an example of a real-world complex network. What type of network model is this? Justify the answer. Describe the general properties and the basic topological properties of this type of network model. You should mention, at least, the basic topological properties related to the size, density and connectivity of the network. You are also expected to explain the general meaning of these topological properties. (10 marks)

   b) Give and explain the basic algorithm for obtaining the type of complex network model mentioned in the previous question. Are there any extensions of the basic algorithm that you have described? If so, explain them. You are expected to give the main differences between all these algorithms. (15 marks)
2. Question 6 (25 marks)

   a) What is synchronisation in complex dynamical networks? What are the main elements to consider in order to establish the synchronisation in a complex dynamical network? What is a dynamical network in this context? (10 marks)

   b) Describe the main model of synchronisation of coupled-dynamical systems in networks in which identical oscillators are non-linearly coupled. You should describe the mathematical model in detail. (15 marks)