Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Symbolic AI

Date: Tuesday 21st May 2013
Time: 09:45 - 11:45

Please answer any THREE Questions from the FOUR Questions provided

This is a CLOSED book examination

The use of electronic calculators is permitted provided they are not programmable and do not store text

[PTO]
1. a) Let the Prolog predicate \( a/3 \) be defined by the single clause

\[
a(X/Y, Y/Z, Z/W).
\]

To what will the variables \( X \), \( Y \) and \( L \) be bound as a result of the following query?

\[
a([\text{red}, \text{yellow}, \text{pink}, \text{green}|X]/X, [\text{orange}, \text{purple}, \text{blue}|Y]/Y, L/[]).
\]

(4 marks)

b) When a file containing the dcg rule

\[
s --> np, vp.
\]

is consulted in Prolog, that rule is converted into an ordinary Prolog clause for the predicate \( s/2 \). Write that clause. (4 marks)

c) Define a Prolog predicate \( \text{myRev}/2 \), which reverses a list in linear time. You should use an 'accumulator', as described in the lectures, or some similar device. (4 marks)

d) Define a predicate \( \text{recRev}/2 \) which reverses a list, and also reverses any lists it may contain, and reverses any lists which those lists may contain, and so on, recursively. Thus:

\[
?- \text{recRev}([1, [2, 3, 4, [5, 6]]], L).
\]

\[
L = [[[6, 5], 4, 3, 2], 1]
\]

For the purposes of this question, you may take a list to be anything which unifies with the term \([X|Y]\). Your predicate should not resatisfy. You will not lose marks for not using accumulators. (4 marks)

e) The Ackermann function \( A(m, n) \), which takes two natural numbers as arguments and yields a natural number as output, is defined as follows.

\[
A(0, n) = n + 1
\]

\[
A(m + 1, 0) = A(m, 1)
\]

\[
A(m + 1, n + 1) = A(m, A(m + 1, n)).
\]

Define a Prolog predicate \( \text{acc}/3 \) which computes this function. (4 marks)
2. a) Translate the following English sentences into first-order logic, using some sensible signature of non-logical primitives.

i) Every artist respects some beekeeper
ii) Every beekeeper is a carpenter
iii) Some artist respects no carpenter.

(5 marks)

b) Put the formulas you gave in your answer to Question 2a in prenex form, and Skolemize.

(5 marks)

c) Put the formulas you gave in your answer to Question 2b in clause form. (You should have five clauses in all.)

(5 marks)

d) Using resolution theorem-proving, derive the empty clause from the clauses you gave in your answer to Question 2c.

(5 marks)

3. a) Give an X-style analysis of the English IPs

i) Every boy does love some girl
ii) Every boy loves some girl

showing how, in the second case, the verb moves from its position in deep structure to join on to the inflection.

(8 marks)

b) Most inhabitants of the planet Numeria speak a language in which the only words are 0 and 1, and the grammatical sentences are strings consisting of some positive number of 0’s followed by the same number of 1’s, or, alternatively, some positive number of 1’s followed by the same number of 0’s. Thus, for example, “0 0 0 0 1 1 1 1”, “1 0” and “1 1 0 0” are all Numerian sentences; however “0 0 1” or “0 0 1 1 0 0” are not. Write a context-free grammar which generates exactly this language.

(6 marks)

c) As a matter of fact, high-caste Numerian speakers know another word, namely 2. Their sentences consist of a positive number of 0s, the same number of 1s and the same number of 2s, always in that order. Write a Prolog definite clause grammar which generates exactly this language. (Warning: you will need to use variables in your dcg rules, in order to keep track of how many 0’s, 1’s or 2’s are present.)

(6 marks)
4. a) Compute the β-reduced forms of the following expressions in the simply-typed λ-calculus, showing your working:

i) \( \lambda p \lambda x [\text{intelligent}(x) \land p(x)] (\lambda y [\text{girl}(y)]) \)

ii) \( \lambda s \lambda x [s (\lambda y [\text{love}(x, y)])](\lambda p [p(john)]) \)

(8 marks)

b) Consider the following semantically annotated context-free grammar:

\[
\begin{align*}
S/\phi(\psi) & \rightarrow \text{NP}/\phi \text{ VP}/\psi \\
\text{NP}/\phi & \rightarrow \text{PropN}/\phi \\
\text{NP}/\phi(\psi) & \rightarrow \text{Det}/\phi \text{ N'}/\psi \\
\text{N'}/\phi(\psi) & \rightarrow \text{Adj}/\phi \text{ N}/\psi \\
\text{N'}/\phi & \rightarrow \text{N}/\phi \\
\text{VP}/\phi(\psi) & \rightarrow \text{V}/\phi \text{ NP}/\psi \\
\text{N}/\lambda x [\text{girl}(x)] & \rightarrow \text{girl} \\
\text{V}/\lambda s \lambda x [s (\lambda y [\text{love}(x, y)])] & \rightarrow \text{loves} \\
\text{PropN}/\lambda p [p(john)] & \rightarrow \text{John} \\
\text{Det}/\lambda p \lambda q [\forall x (p(x) \rightarrow q(x))] & \rightarrow \text{every} \\
\text{Adj}/\lambda p \lambda x [\text{intelligent}(x) \land p(x)] & \rightarrow \text{intelligent} \\
\end{align*}
\]

Draw a picture of the phrase-structure of the sentence

Every intelligent girl loves John.

under this grammar. (This part of the question does not require you to compute the sentence meaning.)

(6 marks)

c) Using your answers to Question 4a, compute the meaning of the sentence given in Question 4b as a formula of first-order logic. Show any other working in full.

(6 marks)