Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Advanced Algorithms II

Date: Thursday 16th May 2013
Time: 09:45 - 11:45

Please answer ONE Question from EACH Section
(a total of THREE Questions from the SIX Questions provided.
Answer all the sub-questions included in the question you have selected)

Use a SEPARATE answerbook for each SECTION

This is a CLOSED book examination

The use of electronic calculators is permitted provided
they are not programmable and do not store text

[PTO]
Section A

This section contains two questions (Question 1 and Question 2). Answer only one of them.

1. Question 1 (25 marks)
   
   Optimisation algorithms follow diverse strategies to find minima or maxima of functions. A popular class of algorithms is based on derivatives of the objective function.

   a) There are two classes of methods for minimising unidimensional functions. Name them and describe one of these types of methods. (12 marks)

   b) Describe one type of multivariate optimisation algorithm based on derivatives (13 marks)
2. Question 2 (25 marks)

Evolutionary algorithms are popular for global optimisation problems. Several variants exist but they follow similar ideas.

a) Describe the two main forces that drive evolutionary algorithms and allow them to achieve minima (or maxima) exploring a large part of parameter space.

(7 marks)

b) Describe the main operators used by evolutionary algorithms.

(7 marks)

c) Genetic programming is a special type of evolutionary algorithm. Describe how it is different from the others.

(6 marks)

d) How is genetic programming similar to how a human creates a program?

(5 marks)
Section B

This section contains two questions (Question 3 and Question 4). Answer only one of them.

1. Question 3 (25 marks)

   In most real complex networks with a large number of nodes the average distance between two arbitrarily chosen nodes is relatively small for the huge size of the network \( N \), with \( N \) the number of nodes. This is known as the small-world property of complex networks.

   a) What type of network model can best capture the small-world property? Explain the general properties and the basic topological properties of this type of network model. You should mention the basic topological properties related to the size, density and connectivity of the network and the corresponding measures for these properties. You are also expected to explain the general meaning of these topological properties.

      (10 marks)

   b) Describe the basic algorithms that can generate the type of complex network model mentioned in the previous question. You are expected to give the main differences between all the algorithms mentioned.

      (15 marks)
2. Question 4 (25 marks)

Multi-agent systems can describe the phenomena of self-organisation and emergence of collective behaviour in groups (for example, flocking or swarming). They differ from dynamical networks, mainly because dynamical networks are typically used to study the synchronisation of coupled oscillators.

a) Describe the main characteristics of the phenomenon of self-organisation in groups in the context of multi-agent systems. (10 marks)

b) Choose one model of swarming or flocking, and describe the main elements and rules of the model chosen. (15 marks)
Section C

This section contains two questions (Question 5 and Question 6). Answer only one of them.

1. Question 5 (25 marks)

a) Give a computer representation of a single precision floating point number (following the IEEE standard) in terms of 4 integers: the mantissa, the base, the precision and the exponent. Explain the difference between the normalised and the denormalised number. Discuss the floating point representation of the number 0. (Hint: Recall the IEEE standard for special numbers). (4 marks)

b) Assume that a computer uses 8-bit representation of floating point numbers, with 4 bits reserved for the mantissa, 3 bits for the exponent and the most significant bit for the sign. If the binary system is used ($b=2$), give a floating point representation $\bar{x}$ of the number $x=0.3$, assuming a simple truncation. (Hint: the exponent bias for this case is 3.). What is the value of the absolute error $e = |\bar{x} - x|$? (9 marks)

c) Write an expression for the bound of the absolute error for the summation $y = x_1 + x_2 + \cdots + x_n$. (Hint: Use graphs to represent the error propagation through the computation for $n = 2$ and $n = 3$, and mathematical induction to prove the general case for an arbitrary $n$). Assume that $0 < x_1 < x_2 < \cdots < x_n$ and that the numbers $x_i$, $i = 1, \ldots, n$ are given exactly (i.e. $r_{x_i} = 0$ for $i = 1, \ldots, n$), and that the errors arise only due to the summation. Based on the obtained expression for $|e'_y|$ conclude which order of performing the summation minimises the error. (12 marks)
2. Question 6 (25 marks)

Consider the following initial value problem:

\[ y' = f(x, y), \quad 0 \leq x \leq 1, \quad y(0) = y_0, \quad (1) \]

where \( f(x, y) \) is a bounded function on \([0, 1]\).

a) Give a formula for a general linear multistep method of order \( k \) for the solution of problem (1). State the difference between explicit and implicit linear multistep methods. (3 marks)

b) Give the formulas for determining the consistency and the order of convergence of a linear multistep method. Give the necessary conditions for consistency and stability of a linear multistep method. (6 marks)

c) Consider the following linear multistep method:

\[ y_{n+3} + ay_{n+2} = \frac{h}{12}(23f_{n+2} - 16f_{n+1} + 5f_n), \quad (2) \]

i) Determine the order of the method (2) as a function of the real parameter \( a \) using the formulas from Part b). (4 marks)

ii) Find the interval of the real parameters \( a \) for which the method is stable. (4 marks)

iii) Consider the application of the method (2) with \( a = -1 \) to the solution of the following initial value problem:

\[ y' = -5x, \quad 0 \leq x \leq 1, \quad y(0) = 1. \quad (3) \]

Adopt the step size \( h = 0.25 \) and the following initial values (given with 3 decimal places): \( y_0 = 1.000, y_1 = 0.8438, y_2 = 0.3750 \). Compute the approximate solutions \( y_3 \) and \( y_4 \) using (2) with 4 decimal places. Compare the computed values \( y_3 \) and \( y_4 \) with the exact (analytical) solution of the problem \( y = 1 - \frac{5x^2}{2} \) at the points \( x = 0.75 \) and \( x = 1 \), respectively. What can be concluded? (8 marks)