Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Advanced Algorithms II

Date: Tuesday 27th May 2014
Time: 09:45 - 11:45

Please answer ONE Question from EACH Section
(a total of THREE Questions from the SIX Questions provided.
Answer all the sub-questions included in the question you have selected)

Use a SEPARATE answerbook for each SECTION

This is a CLOSED book examination

The use of electronic calculators is permitted provided they are
not programmable and do not store text
Section A

This section contains two questions (Question 1 and Question 2).

Answer only one of them.

1. Question 1 (25 marks)

   a) Give a computer representation of a floating point number in terms of the mantissa, the base, the precision and the exponent. Explain the difference between a normalised and a denormalised number. (3 marks)

   b) Suppose that a model computer stores floating point numbers in registers of length 4, where the 2 most significant bits are used to register the exponent and the two least significant bits are used to register the normalised mantissa. It is assumed that there is no implicit storage of the leading non-zero bit of the mantissa. In addition, there is no sign bit, and all the numbers are assumed to be positive. The computer uses binary numbers ($b = 2$).

      i. Write all the numbers that can be represented as floating point numbers on this model computer. Assume that the exponent range is $e_{\text{min}} = -2$, $e_{\text{max}} = 1$ and that the bias is 2. (8 marks)

      ii. Write decimal equivalents ($b = 10$) of the numbers from Part i. (3 marks)

      iii. Perform the computation $z = x_1 + x_2 + x_3$ where $x_1 = \frac{5}{32}$, $x_2 = \frac{3}{16}$, $x_3 = \frac{17}{64}$ using the floating point representation of these numbers on the model computer. In this context, if a number $x_i$ cannot be represented exactly, the closest representable floating point number $\bar{x}_i$ is used instead. Assume in your answer that rounding of the mantissa is used when representing a real number as a floating point number on a model computer. (5 marks)

      iv. Analyse the error propagation in the computation from part iii) using graphs. In this context, assume that the relative errors of the initial data $|r_i| = |e_i|/|x_i|$, and the unit roundoff $u$ are given, and draw the graph representing the computation of $\bar{z}$. Based on this graph, estimate the total relative error in computing $\bar{z}$ on the model computer. (6 marks)
2. Question 2 (25 marks)

Consider the following initial value problem:

\[ y' = f(x, y), \quad a \leq x \leq b, \quad y(a) = y_a. \quad (1) \]

a) Give a formula for a general linear multistep method of order \( k \) for the solution of the problem (1). In this context, explain the difference between explicit and implicit linear multistep methods. (3 marks)

b) Give the formulas for determining the order of convergence of a linear multistep method. Give the necessary conditions for consistency and stability of a linear multistep method. (5 marks)

c) Consider the following linear multistep methods:

\[ y_{n+2} - y_{n+1} = \frac{h}{3} (3f_{n+1} - 2f_n), \quad n = 0, 1, \ldots, \quad (2) \]
\[ y_{n+2} - y_{n+1} = \frac{h}{2} (3f_{n+1} - f_n), \quad n = 0, 1, \ldots. \quad (3) \]

i. Determine the order of the methods (2) and (3). Are these methods consistent? (5 marks)

ii. Consider the application of the methods (2) and (3) to the following initial value problem:

\[ y' = x^2 + y, \quad 1 \leq x \leq 2, \quad y(1) = 1. \quad (4) \]

Using the step size \( h = 0.25 \) and the following initial values (given with 3 decimal places) \( y_0 = 1.000, \ y_1 = 1.642 \). Compute the approximate solutions \( y_2, y_3, \) and \( y_4 \) using both methods (2) and (3). Round all computations to 3 decimal places. (7 marks)

iii. Compare the numerical solutions computed in ii) with the exact solution given by \( y(x) = 6e^x - x^2 - 2x - 2 \). What can be concluded? (5 marks)
3. Question 3 (25 marks)

There are many different optimisation algorithms, each one following a specialised strategy to localize optima of functions. While it is common to discuss their details, it is also important to recognise that they are fundamentally similar. This type of abstraction helps identify parts that are common to all algorithms which will make implementations more efficient.

a) A large class of algorithms is based on a single candidate solution, while others are based on populations of solutions. Summarise, in general terms, the operation of all single candidate solution algorithms.  (4 marks)

b) All iterative algorithms need criteria for stopping and avoiding infinite loops. Describe one stopping criterion that can be applied for all numerical optimisation algorithms. Contrast with a common stopping criterion used in derivative-based methods.  (9 marks)

c) Some algorithms are only able to locate local minima, while others are able to locate global minima. Describe the difference between local and global minima.  (4 marks)

d) Describe general characteristics of global optimisation algorithms that allow them to escape local minima. Provide examples.  (8 marks)
4. Question 4 (25 marks)

Population-based algorithms are popular particularly for global optimisation problems. Perhaps the most studied and widely used of these are the evolutionary algorithms (EAs).

a) Two fundamental operators in EAs are reproduction and selection. Describe the loop of one algorithm where reproduction happens before selection, with overlapping generations. Describe the size of the population at each step. (9 marks)

b) Describe an alternative algorithm to the above, where selection takes place before reproduction and there are no overlapping of generations. Describe the size of the population at each step. (9 marks)

c) The mutation operator is used in all EAs to introduce variation. Discuss how mutation is implemented when the genome is represented in a) binary, b) floating point, and c) parse trees (for genetic programming) (7 marks)
Section C

This section contains two questions (Question 5 and Question 6).

Answer only one of them.

5. Question 5 (25 marks)
Consider the World Wide Web network, where nodes are web pages and links are the links between pages. This is an example of a real-world complex network. At every level of the network, there are some pages with many links into or out of them, and many other pages with few links into or out of them. That is, there is a very large number of pages with a low degree, and a very small number of pages that have a high degree.

a) What type of network model can best capture the structure of the World Wide Web? Explain the general properties and the basic topological properties of this type of network model. You should describe the basic topological properties related to the size, density and connectivity of this network and the corresponding measures for these properties. You are also expected to explain the meaning of these properties in the context of the World Wide Web network. (10 marks)

b) Describe the basic algorithm for obtaining the type of complex network model mentioned in question 5.a). Are there any extensions of the basic algorithm that you have described? If so, explain them. (15 marks)
6. Question 6 (25 marks)

Complex dynamical networks are typically used to study the synchronisation of coupled oscillators.

a) What is synchronisation in complex dynamical networks? What are the main elements to consider in order to establish synchronisation in a complex dynamical network? (10 marks)

b) Among the models of synchronisation of coupled-dynamical systems in networks that we have studied in class, what is the one that best describe the synchronisation of non-identical oscillators? (2 marks)

c) Explain the main elements and properties of the model mentioned in question 6.b). You are expected to describe how oscillators are coupled and how the level of synchronisation achieved by the coupled oscillators is quantified in this model. (13 marks)