Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

AI and Games

Date: Wednesday 3rd June 2015
Time: 14:00 - 16:00

Please answer any THREE Questions from the FOUR Questions provided

Use a SEPARATE answerbook for each QUESTION

This is a CLOSED book examination

The use of electronic calculators is NOT permitted

[PTO]
1. a) In the context of this course, define a game. (3 marks)

b) Suppose it is announced that the game of chess is solved. Give 3 things that this could possibly mean. (3 marks)

c) The figure below shows a game tree for a two-player, zero-sum game. The square nodes are the MAX player and the round nodes are the MIN player. Terminal nodes are labeled by their outcomes: win (W) or loss (L), which can be taken as 1 and 0 respectively. The branches are labelled b1 – b25.

   ![Game Tree Diagram]

i) What is the value of the root node? Justify your answer. (2 marks)

ii) What branches of the subtree do not have to be evaluated? Assume that child nodes are evaluated from left to right. (3 marks)

d) Is it always the case for a two-player, zero-sum game in normal form to have an equilibrium of the form,

\[
\max_i \min_j a_{ij} = a_{i^* j^*} = \min_j \max_i a_{ij}.
\]

If not, give a counter-example. (3 marks)

e) What follows is a two-player game in normal form. What is its equilibrium and why. (4 marks)

   \[
   \begin{array}{ccc}
   & A & B & C \\
   a & 2, -2 & 0, 0 & 1, -1 \\
   b & 0, -0 & -5, 5 & 2, -2 \\
   c & 4, -4 & 5, -5 & 3, -3 \\
   \end{array}
   \]

f) In a large game, what is the role of a heuristic evaluation function? (2 marks)
2. Consider the game Kalah, with seven wells per player and seven stones in each well.

   a) How would you classify this game? What is known about its possible solutions for this and smaller versions of the game?  

   (4 marks)

   b) Give an overview of the program your group wrote to play Kalah. Describe the main parts of the program and how they fit together?  

   (6 marks)

   c) Sketch an approach to a program playing this game which significantly differs from the one you chose.  

   (4 marks)

   d) What do you consider to be the greatest strength of your group’s program? How do you think that affected your program’s performance? Justify your answer.  

   (3 marks)

   e) What would have been the one idea that could have considerably strengthened your program? Justify your response.  

   (3 marks)
3. a) Consider a three-person three level Stackelberg game as follows:

- The first player, $P_L$, whose strategy and strategy space are $u_L$ and $U_L$, is the leader; The second player, $P_{FL}$, whose strategy and strategy space are $u_{FL}$ and $U_{FL}$, is a follower of the first player but a leader to the third player; The third player, $P_F$, whose strategy and strategy spaces are $u_F$ and $U_F$, is a follower of both the first and second players.
- The payoff functions for $P_L$, $P_{FL}$ and $P_F$ are $J_L(u_L, u_{FL}, u_F)$, $J_{FL}(u_L, u_{FL}, u_F)$, and $J_F(u_L, u_{FL}, u_F)$ respectively and each player’s goal is to maximise his payoff function.
- The rule of playing is that, firstly, the first player, $P_L$, announces his strategy $u_L$; after knowing the first player strategy, the second player, $P_{FL}$, announces his strategy $u_{FL}$; after knowing both the first and second players’ strategies, the third player, $P_F$, select his best response strategy.

For the above three-person Stackelberg game, what is the definition of a Stackelberg strategy (i.e., Stackelberg equilibrium)? (6 marks)

b) Find a Stackelberg strategy for the following Stackelberg game:

- There are two players in which L is the leader and F is the follower;
- The strategy spaces for the leader and the follower are continuous ones as $U_L = [0, +\infty)$ and $U_F = [0, +\infty)$;
- The payoff functions for the leader and the follower are
  
  $J_L(u_L, u_F) = -u_L^2 - 2u_Lu_F - u_F^2 + 8u_L + 5u_F + 10$
  
  $J_F(u_L, u_F) = -4u_L^2 + 4u_Lu_F - u_F^2 + 8$

in which $u_L \in U_L$ is the leader’s strategy and $u_F \in U_F$ is the follower’s strategy.

Note. 9 marks are for the step by step process, and 1 mark is for the correct answer. (10 marks)

c) Find a Stackelberg strategy for the following Stackelberg game:

- There are two players in which L is the leader and F is the follower;
- The strategy spaces for the leader and the follower are finite ones and contain only two strategies as $U_L = \{0, 1\}$ and $U_F = \{0, 1\}$ (i.e., the leader or the follower can only choose his strategy either as 0 or 1);
- The payoff functions for the leader and the follower are

<table>
<thead>
<tr>
<th>Leader</th>
<th>Follower</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(3,5) (4,6)</td>
</tr>
<tr>
<td>1</td>
<td>(2,4) (1,3)</td>
</tr>
</tbody>
</table>

(4 marks)
4. a) Briefly explain what is mechanism design. Why is mechanism design sometimes also called reverse game theory? (3 marks)

b) Consider the following single-item auction game: There is one item for sale and there are \( n \) bidders with unknown private values for the item, \( v_1, \ldots, v_n \) and \( v_i \neq v_j (i \neq j) \). Each bidder’s objective is to maximize his utility \( u_i = v_i - p \), where \( p \) is the price paid to the item if the bidder wins. Assume that the second-price auction is the chosen mechanism by the seller whose goal is to maximise the social surplus function \( S(x_1, \ldots, x_n) = \sum_{i=1}^{n} v_i x_i \), where \( x_i \) is 1 if bidder \( i \) wins and 0 if loses, and only one bidder can win the item. For the given auction game and the chosen mechanism, now answer the following questions:

i) How to define Incentive Compatible? (1 mark)

ii) How to define Dominant-Strategy Incentive-Compatible? (1 mark)

iii) Assume that each bidder bids his truthful value. Why does the chosen auction mechanism maximise the social surplus function? (3 marks)

iv) Why is the above social surplus maximisation problem solvable by a polynomial time algorithm? (2 marks)

v) For bidder \( i \), why is it true that bidding the true valuation \( v_i \) is his dominant strategy? (8 marks)

vi) From an application point of view, what is the main advantages of the second-price auction in comparison with the first-price auction? (2 marks)