Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Advanced Algorithms II

Date: Friday 29th May 2015
Time: 14:00 - 16:00

Please answer ONE Question from EACH Section
(a total of THREE Questions from the SIX Questions provided.
Answer all the sub-questions included in the question you have selected)

Use a SEPARATE answerbook for each SECTION

This is a CLOSED book examination

The use of electronic calculators is permitted provided they are
not programmable and do not store text

[PTO]
Section A

This section contains two questions (Question 1 and Question 2).

Answer only one of them.

1. Question 1 (25 marks)

a) Describe the double precision format used for registering floating point numbers on a computer according to the IEEE 754 standard. Give the number of bits used to store the mantissa, the exponent and the sign in this case. Give the approximate range (both in binary and decimal terms) of real numbers that can be represented in this format. What is the exponent bias in this case? In your answer assume that the mantissa is normalised and has a format \( m = 0.1d_2d_3\ldots d_n \), where \( d_i \in \{0, 1\} \), \( i = 2, \ldots, n \) are the binary digits. (4 marks)

b) Suppose that a model computer which uses a binary system \((b = 2)\) has registers of the length 4 and that floating point numbers are represented in the following format:

\[
\begin{array}{c|c|c|c}
3 & 2 & 1 & 0 \\
\hline
\text{sign} & \text{exponent} & \text{mantissa} \\
\end{array}
\]

where the sign bit is 0 for positive and 1 for negative numbers. The exponent is registered with the bias 2 and the mantissa is normalised, i.e. it is in the format \( m = 0.1d_2 \), where \( d_2 \in \{0, 1\} \). We assume that the leading digit in the mantissa is implicit and only \( d_2 \) is actually stored.

i) Write all the numbers that can be represented on this computer. In this context, for each representable number give the 4-digit floating point representation and its equivalent meaning in the format \( \bar{x} = \pm 0.d_1d_2 \times 2^e \). (6 marks)

ii) Give decimal equivalents of the numbers that can be represented on the model computer. (6 marks)

iii) Perform the computation \( z = x_1 + x_2 + x_3 \), where \( x_1 = -\frac{5}{32} \), \( x_2 = \frac{3}{16} \), \( x_3 = \frac{17}{64} \) using the floating point representations of these numbers \( \bar{x}_1 \), \( \bar{x}_2 \) and \( \bar{x}_3 \) on a model computer introduced in Part b). In this context, if a number cannot be represented exactly, the closest representable floating point number is used instead. Assume in your answer that rounding of the mantissa is used when representing a real number as a floating point number on the model computer. (9 marks)
2. Question 2 (25 marks)

Consider the following initial value problem:

\[ y' = f(x, y), \quad a \leq x \leq b, \quad y(a) = y_a \]  

(1)

where \( f(x, y) \) is a bounded function on \([a, b]\).

a) Give a formula for a general linear multistep method of order \( k \) for the numerical solution of the problem (1). In this context, explain the main difference in computational cost between explicit and implicit methods. 

(4 marks)

b) Give the formulas for determining the order of convergence of a linear multistep method. Give the necessary conditions for consistency and stability of a linear multistep method.

(5 marks)

c) Consider the following linear multistep method:

\[
\frac{1}{3}y_{n+3} + \frac{1}{2}y_{n+2} - y_{n+1} + \frac{1}{6}y_n = hf_{n+2}, \quad n = 0, 1, \ldots,
\]  

(2)

where \( h \) is the discrete step size and \( f \) is the right hand side of (1).

i) Determine the order of method (2). Is the method consistent? Write the first characteristic polynomial for method (2) and, based on its zeros, discuss the stability of the method. 

(6 marks)

ii) Consider the application of the method (2) to the solution of the following problem:

\[ y' = -10y, \quad 0 \leq x \leq 1, \quad y(0) = 1. \]  

(3)

Adopt the step size \( h = 0.2 \) and the following initial values given with 4 decimal places: \( y_0 = 1.0000, y_1 = 0.1353, y_2 = 0.0183 \). Compute the approximate solutions \( y_3, y_4 \) and \( y_5 \).

(6 marks)

iii) Compare the numerical solutions \( y_3, y_4, \) and \( y_5 \) computed in part ii) with the exact solution given by \( y(x) = e^{-10x} \). What can be concluded? 

(4 marks)
3. Question 3 (25 marks)

Optimisation algorithms are used to find minima or maxima of functions. There are two main strategies to minimise functions of a single variable, while the minimisation of multivariate functions can be carried out through many diverse approaches.

a) Describe the interval reduction method for minimisation of univariate functions. (13 marks)

b) How would you maximise a function $f(x)$ if you have access only to algorithms for minimisation of functions? (3 marks)

c) A large class of algorithms for the minimisation of multivariate functions are based on a single candidate solution, while others are based on populations of solutions. Summarise, in general terms, the operation of all single candidate solution algorithms. (4 marks)

d) Some algorithms are able to locate only local minima, while others are able to locate global minima. Describe the difference between local and global minima. (5 marks)
4. Question 4 (25 marks)

Evolutionary algorithms (EAs) are widely used to solve global optimisation problems.

a) Describe the main operators used by evolutionary algorithms. (8 marks)

b) Describe what are the main characteristics of evolutionary algorithms that make them efficient at escaping local minima, including the dicotomy of diversity versus selection. (7 marks)

c) Mutation is used in all EAs to introduce variation. Describe how mutation is implemented when the candidate solutions are represented in a) binary, b) floating point, and c) parse trees (for genetic programming). (7 marks)

d) Genetic programming (GP) is one type of EA that differs considerably from all the others. Describe the main differences between GP and other EAs. (3 marks)
Section C

This section contains two questions (Question 5 and Question 6).

Answer only one of them.

5. Question 5 (25 marks)
Consider a network of Hollywood actors, where nodes are actors and a link is established between two actors if they have appeared in the same movie at some point in their careers. This is an example of a real-world complex network. In this network, there are some actors with many links, and many other actors with few links. That is, there is a very large number of actors with a low degree, and a very small number of actors that have a high degree.

a) What type of network model can best capture the structure of the network of actors of Hollywood? Explain the general properties and the basic topological properties of this type of network model. You should describe the basic topological properties related to the size, density and connectivity of this network and the corresponding measures for these properties. You are also expected to explain the meaning of these properties in the context of the network of Hollywood actors. (10 marks)

b) Describe the basic algorithm for obtaining the type of complex network model introduced in 5.a). (5 marks)

c) Explain the two main extended versions of the basic algorithm that you have described in 5.b). (10 marks)
6. Question 6 (25 marks)

Complex dynamical networks are used to study the synchronisation of coupled oscillators.

a) What is synchronisation in complex dynamical networks? What are the main elements to consider in order to establish synchronisation in a complex dynamical network? (10 marks)

b) Among the models of synchronisation of coupled-dynamical systems in networks that we have studied in class, what is the one that best describes the synchronisation of identical oscillators that are non-linearly coupled? (5 marks)

c) Explain the main elements and properties of the model you chose in 6.b). You are expected to describe how oscillators are coupled. (10 marks)