Quantum Computing

Date: Tuesday 2nd June 2015
Time: 14:00 - 16:00

Please answer any THREE Questions from the FIVE Questions provided

This is a CLOSED book examination
The use of electronic calculators is NOT permitted
1.  a) Write down the Hardamard matrix $H$. (3 marks)  

b) Is $H$ Hermitian? (2 marks)  
c) Is $H$ unitary. (2 marks)  
d) How are the coefficients $\lambda_k$ calculated in the expansion $|x\rangle = \sum_k \lambda_k |k\rangle$ of an arbitrary vector $|x\rangle$ when the $|k\rangle$ form an orthonormal basis? (3 marks)  
e) Define an eigenvalue of a $2 \times 2$ matrix. (2 marks)  
f) Define an eigenvector of a $2 \times 2$ matrix. (2 marks)  
g) What are the eigenvalues and eigenvectors of $H$? (6 marks)  

2.  a) What is a state space of a quantum system? (2 marks)  
b) How is the state space of a composite system related to the state spaces of its individual components? (2 marks)  
c) How does an isolated quantum system evolve from a time $t_0$ to a time $t_1$? (2 marks)  
d) What is an observable? (2 marks)  
e) What is an observation? (2 marks)  
f) Distinguish carefully between a measurement and an observation in quantum physics. (2 marks)  
g) State and prove the No-Cloning Theorem. (4 marks)  
h) Explain the difference between cloning and basis copying. Illustrate your answer on the state $a|0\rangle + b|1\rangle$ of $Q$. Write down a basis copying operator for the state space $Q$. (4 marks)
3. a) Write down the 1-quibit gates for the $I$, $X$, $Y$, $Z$, $\Phi_\theta$ operators. (3 marks)

b) Describe the Toffoli gate. (3 marks)

c) Calculate the effect of the operator $\Phi_{\phi+\pi/2}^H\Phi_\theta^H$ on the $|0\rangle$ state. (4 marks)

d) Explain carefully what the following quantum circuit does. Your answer should make clear how each basis vector is transformed. (10 marks)

4. a) Derive the CHSH inequality. (4 marks)

b) In $Q \otimes Q$ is the state

$$\sqrt{\frac{3}{5}} |00\rangle + \sqrt{\frac{2}{5}} |11\rangle$$

separable or entangled? Justify your answer. (2 marks)

c) Explain the Bell basis. (4 marks)

d) Give an account of the superdense coding algorithm. (10 marks)

5. Give a detailed account of Grover’s algorithm. (20 marks)

END OF EXAMINATION