COMP34120

Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

AI and Games

Date: Tuesday 7th June 2016

Time: 14:00 - 16:00

Please answer any THREE Questions from the FOUR Questions provided

Use a SEPARATE answerbook for each QUESTION

This is a CLOSED book examination

The use of electronic calculators is NOT permitted

[PTO]

1. a) If it is announced that the $(7,7)$ Kalah is *solved*, give 3 things that this could possibly mean. (3 marks)

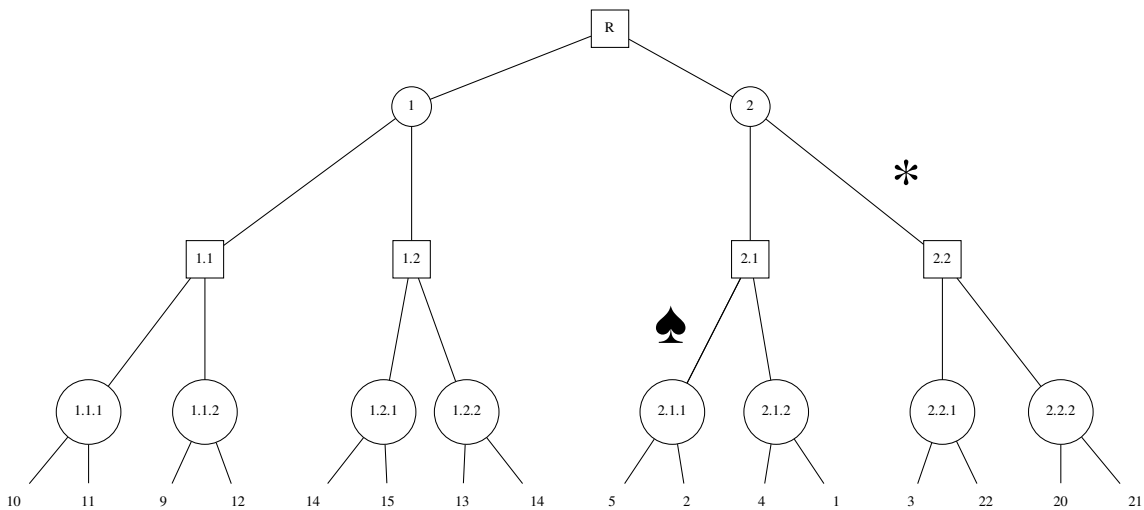
b) The following is a two-player, zero-sum game in normal form.

		Alice			
		A	*B*	*C*	*D*
	a	4	3	2	5
Bob	*b*	3	2	1	4
	c	2	1	0	3

i) Find the equilibrium of this game using the minimax approach. (3 marks)

ii) Find the equilibrium of this game using another approach. (2 marks)

c) The figure below shows a game tree for a two-player, zero-sum game. The square nodes are the `MAX` player and the round nodes are the `MIN` player. Terminal nodes are labelled by their outcomes, which is a value.



i) What is the value of the root node? Justify your answer. (2 marks)

ii) What branches of the subtree do *not* have to be evaluated? Assume that child nodes are evaluated from left to right.
To label a branch, list the parent of that branch and the direction of the child: 'Left' or 'Right'. For example, the branch marked with the asterisk ($*$) would be 2 'Right' which would eliminate 2.2, 2.2.1. and 2.2.2. The branch marked with the spade symbol (♠) would be 2.1 'Left', which would eliminate 2.1.1. (5 marks)

d) What follows is a two-player zero-sum game in normal form.

		Cassie	
		A	B
Robert	a	3	1
	b	1	3

Find or guess a Nash equilibrium for this game, and show it is a Nash equilibrium.
(5 marks)

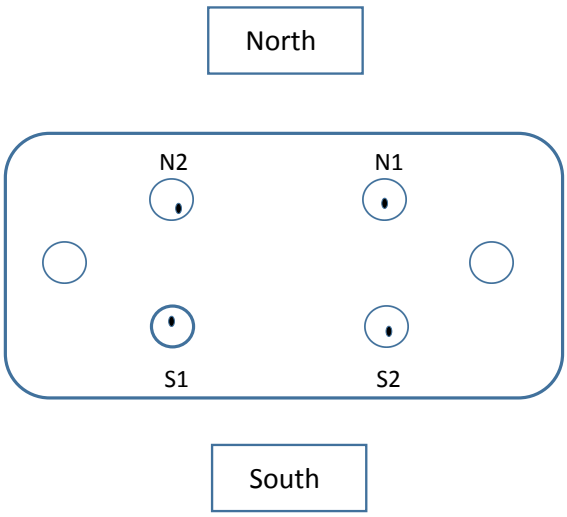
2. Consider the game Kalah, with seven wells per player and seven stones in each well.

   a) How would you classify this game? What is known about its possible solutions for this and smaller versions of the game? (4 marks)

   b) Give an overview of the program your group wrote to play Kalah. Describe the main parts of the program and how they fit together. (6 marks)

   c) If your group used primarily a learning-based approach, describe how to apply a non-learning approach to some aspect of the game AI. Likewise, if your group used primarily a non-learning approach, describe a learning-based method for the game of Kalah. (4 marks)

   d) Consider the $(2, 1)$ version of Kalah. That is, there are two 'houses' on each side of the board, and each house starts with one seed. A sketch of the game board at the start of the game is below. Player 1 is SOUTH and goes first.



      i) Solve the $(2, 1)$ version of Kalah *without the pie rule*. Which player can force which outcome and with what strategy? (3 marks)

      ii) What is the solution with the pie rule? Remember, with the pie rule, the first player on its first play cannot get an extra move, even if the last seed it deposits is in its scoring well. (3 marks)

3. a) In a two-person Stackelberg game with imperfect information, the follower's reaction function, $y = R(x)$, often has to be learned from historical data $\{y(t), x(t)\}$ $(t = 1, 2, ..., T)$ and the best estimated reaction function, $y = \hat{R}(x, \theta)$, is obtained by minimising the sum of square errors as below:

$$min_{\theta} \sum_{t=1}^{T} \{y(t) - \hat{R}[x(t), \theta]\}^2$$

Now answer the following questions:

i) For the moving window approach, give the revised sum of square errors to be minimised in order to obtain the best estimated reaction function and briefly explain its meaning. (2 marks)

ii) For the recursive least square approach with the forgetting factor, give the revised sum of square errors to be mimimised in order to obtain the best estimated reaction function and briefly explain its meaning. (2 marks)

iii) Comparing with the moving window approach, what are the main advantages of the recursive least square approach? (3 marks)

b) Consider the following Stackelberg game:

- There are two players in which L is the leader and F is the follower, and the strategy spaces for the leader and the follower are U_L and U_F respectively;
- The payoff functions for the leader and the follower are

$$J_L(u_L, u_F) = -4u_L^2 + 4u_L u_F - u_F^2 + 2u_L + 10$$
$$J_F(u_L, u_F) = -u_L^2 + 2u_L u_F - u_F^2 - 2u_L + 2u_F + 10$$

in which $u_L \in U_L$ is the leader's strategy and $u_F \in U_F$ is the follower's strategy.

Assume that the follower's strategy space is a continuous one given as $U_F = [0, +\infty)$. Now for each of the following cases, find a Stackelberg strategy:

i) The leader's strategy space $U_L = [0, +\infty)$. (7 marks)

ii) The leader's strategy space $U_L = [0, 1]$. (3 marks)

iii) The leader's strategy space $U_L = \{0, 1, 4, 5\}$ (that is, U_L is a discrete strategy space which includes only 4 numbers) (3 marks)

Note. For each case above, 1 mark is for the correct answer and the other marks are for the step by step reasoning and process.

4. a) Mechanism design involves three basic elements: players or agents, mechanism designer or principle, and mechanism design task or problem. Please briefly explain each of these elements in the context of mechanism design. (6 marks)

   b) Consider the following single-item auction game: There is one item for sale and there are 2 bidders whose private valuations for the item are v_1 and v_2 respectively. Each bidder's objective is to maximize his own utility $u_i = v_i - p$ $(i = 1,2)$, where p is the price paid to the item if a bidder wins. For this auction game, answer the following questions:

      i) Assume that the auctioneer's objective is to maximize social surplus and the 2nd price auction is chosen as the mechanism. Then what is the best bidding strategy for each bidder? Is the 2nd price auction an Incentive-Compatible mechanism? (2 marks)

      ii) Assume that v_1 and v_2 are random variables with uniform distributions, the auctioneer's objective is to maximize social surplus and the 1st price auction is chosen as the mechanism. Then what is the best bidding strategy for each bidder? Is the 1st price auction an Incentive-Compatible mechanism? (2 marks)

      iii) If the auctioneer's objective is to maximize revenue, does the 2nd price auction always generate more or at least the same revenue as the first price auction and why? (3 marks)

      iv) If the auctioneer's objective is to maximize revenue,what extra mechanism can be introduced to gain more revenue? (1 mark)

   c) Consider the following multi-item auction game: There are two items: Item 1 and Item 2, which include 20 and 10 identical products respectively. There are three bidders: Bidders 1, 2, and 3, whose valuating per product is 12, 10 and 1 respectively. Assume that the auctioneer's objective is to maximise the social surplus function and his designed allocation rule is to allocate Item 1 to the highest bidder and Item 2 to the 2nd highest bidder. Now answer the following questions:

      i) Briefly describe what is a monotone allocation rule and answer whether the above allocation rule is a monotone one. (2 marks)

      ii) For the given allocation rule, explain why the extended 2nd pricing rule (that is, the highest bidder pays the 2nd highest bidder's bid and the 2nd highest bidder pays the 3rd highest bidder's bid) cannot ensure DSIC (i.e., Dominant Strategy Incentive Compatible). (2 marks)

      iii) Whether there exists a pricing rule which can ensure DSIC? If yes, please give such a pricing rule. (2 marks)

END OF EXAMINATION