COMP36212

Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Advanced Algorithms II

Date: Wednesday 1st June 2016

Time: 09:45 - 11:45

Please answer ONE Question from EACH Section
(a total of THREE Questions from the SIX Questions provided.
Answer all the sub-questions included in the question you have selected)

Use a SEPARATE answerbook for each SECTION

This is a CLOSED book examination

The use of electronic calculators is permitted provided they
are not programmable and do not store text

[PTO]

Section A

This section contains two questions (Question 1 and Question 2).

Answer only one of them.

1. Question 1 (25 marks)

   a) Give a computer representation of a single precision floating point number, according to the IEEE standard, in terms of four integers: the mantissa m, the base b, the precision n, and the exponent e. Explain the difference between a normalised and a denormalised number. Give a single precision representation of the special values $\pm\infty$. (4 marks)

   b) Assume that a model computer uses 8-bit representation of floating point numbers, with 4 least significant bits reserved for the mantissa, the next 3 bits for the exponent, and the most significant bit for the sign. If the binary system is used ($b = 2$), and the normalised format of the mantissa with an explicit storage of the leading bit 1 is assumed, give the floating point representations $\bar{x}_1$ and $\bar{x}_2$ of the decimal numbers $x_1 = 0.3$ and $x_2 = -0.29$ on the described model computer. Assume in your answer that rounding of the floating point numbers is used when discarding the binary digits in the mantissa. (7 marks)

   c) Calculate the number of significant digits in the binary floating point numbers $\bar{x}_1$ and $\bar{x}_2$ from Part b). Perform the addition $\bar{z} = \bar{x}_1 + \bar{x}_2$ with the binary floating point numbers, and estimate the number of significant digits in $\bar{z}$ (Hint: use the formula to calculate the loss of significant digits when the floating point addition is performed). Translate the floating point number $\bar{z}$ back to the decimal system and compare it to the exact value $z = x_1 + x_2 = 0.01$. (14 marks)

2. Question 2 (25 marks)

Consider the following initial value problem:

$$y' = f(x,y), \qquad 0 \le x \le 1, \quad y(0) = y_0 \tag{1}$$

where $f(x,y)$ is a bounded function on $[0,1]$.

a) Give a formula for a general linear multistep method of order k for the numerical solution of the problem (1). What is the difference between explicit and implicit linear multistep methods? (5 marks)

b) Consider the following linear multistep method:

$$y_{n+3} = y_{n+2} + \frac{h}{12}(23f_{n+2} - 16f_{n+1} + 5f_n)\,. \tag{2}$$

i) Determine the order of the method (2). Is the method consistent? (5 marks)

ii) Examine the stability of the linear multistep method (2). (5 marks)

iii) Apply the method (2) to the solution of the harmonic oscillator problem:

$$y'' = -y, \quad y(0) = 1, \quad y'(0) = 1\,. \tag{3}$$

(Hint: The problem (3) can be rewritten as a system of two first order initial value problems by introducing an auxiliary function $z = y'$ and noting that y'' $= (y')'$.) Adopt the step size $h = 0.25$ and the following initial values given with 4 decimal places:

n	0	1	2
y_n	1.0000	1.2163	1.3570
z_n	1.0000	0.7215	0.3982

Compute the approximate solutions y_3, z_3, y_4 and z_4 with 4 decimal places using (2). Compare the computed values to the exact solutions of the problem given by $y = \cos(x) + \sin(x)$ and $z = -\sin(x) + \cos(x)$. (10 marks)

Section B

This section contains two questions (Question 3 and Question 4).

Answer only one of them.

3. Question 3 (25 marks)

   Nonlinear optimisation consists of finding optima of mathematical functions that may depend on several variables (multidimensional). There are many and diverse approaches to search for optima of nonlinear functions.

   a) For unidimensional functions there are two common approaches: interpolation and interval reduction. Describe the interval reduction approach in a clear way such that someone could implement code based on your description. (9 marks)

   b) Apart from unidimensional functions, the interval reduction algorithm is also useful also to minimise multidimensional functions. Explain how gradient-based minimisation algorithms work in general, and how they make use of interval reduction. (7 marks)

   c) Direct search algorithms do not make use of derivatives. An example is the Nelder-Mead algorithm, which keeps a memory of the $n+1$ previously visited locations (coordinates) of the function, where n is the dimensionality of the objective function. Describe how the algorithm proceeds from one iteration to the next. (9 marks)

4. Question 4 (25 marks)

   There are many types of optimisation algorithms inspired by natural phenomena, from physics to biology.

   a) Simulated annealing is an algorithm that is based on the process of crystal formation. Describe the overall strategy used by this algorithm to find global minima. (10 marks)

   b) Evolutionary algorithms are based on how biological species evolve. Describe what are the main characteristics of evolutionary algorithms that allow them to escape local minima. (6 marks)

   c) Genetic programming is an evolutionary algorithm that is used to evolve functions or programs. Describe the most common data structure used to represent functions or programs in genetic programming. (9 marks)

Section C

This section contains two questions (Question 5 and Question 6).

Answer only one of them.

5. Question 5 (25 marks)

Consider the air-traffic network of airports in Europe, where nodes are airports and a link is established between two airports if there is a flight that connects both airports. This is an example of a real-world complex network. In this network, there are some airports with many links, and many other airports with few links. That is, there is a large number of airports with a low node degree, and a relatively small number of airports that have a high node degree.

a) What type of network model can best capture the structure of this air-traffic network of airports? Explain the general properties of this type of network model, including a description of the basic topological properties related to the size, density, and connectivity of this network, and the corresponding measures for these properties. Explain the meaning of these properties in the context of this air-traffic network of airports. (10 marks)

b) Describe the basic algorithm for obtaining the type of complex network model introduced in 5.a). (5 marks)

c) Explain the two extended versions of the basic algorithm that you have described in 5.b). (10 marks)

6. Question 6 (25 marks)

   Complex dynamical networks are used to study the synchronisation of coupled oscillators.

   a) What is synchronisation in complex dynamical networks? What are the main elements to consider in order to establish synchronisation in a complex dynamical network? (10 marks)

   b) Among the models of synchronisation of coupled dynamical systems in networks that we have studied in class, what is the general model that describes the synchronisation of identical oscillators that are linearly coupled? Describe the main types of linear coupling between oscillators used in these networks. (5 marks)

   c) Explain the main elements and properties of the model you mentioned in 6.b). Choosing one of the types of linear coupling between oscillators that you mentioned in 6.b), describe how oscillators are coupled and the mathematical model in detail. (10 marks)

END OF EXAMINATION