COMP39112

Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Quantum Computing

Date: Wednesday 8th June 2016

Time: 14:00 - 16:00

Please answer any THREE Questions from the FIVE Questions provided

This is a CLOSED book examination

The use of electronic calculators is NOT permitted

[PTO]

1. a) For a column vector $|x\rangle = [x_0 \ldots x_n]^T$, how is the norm $\|x\|$ defined? (2 marks)

   b) What does the phrase "$|x\rangle$ and $|y\rangle$ are orthogonal" mean? (2 marks)

   c) Prove the following member of the triangle inequality family:
   $\|x\| - \|y\| \leq \|x - y\|$ (4 marks)

   d) Define hermitian and unitary matrices. (3 marks)

   e) How are hermitian and unitary matrices related? (3 marks)

   f) State the properties of the eigenvalues of hermitian and unitary matrices. (2 marks)

   g) If $|0_H\rangle$ and $|1_H\rangle$ are the Hadamard basis vectors, prove that
   $|0\rangle\otimes|0\rangle + |1\rangle\otimes|1\rangle = |0_H\rangle\otimes|0_H\rangle + |1_H\rangle\otimes|1_H\rangle$ (4 marks)

2. a) Describe the axioms of quantum mechanics relating to: (i) the state space of a system, (ii) the state space of a composite system versus its components, (iii) the evolution of an isolated system over a period of time. (5 marks)

   b) Describe carefully the process of quantum measurement. (5 marks)

   c) Write down as succinctly as you can all possible binary products of the Pauli spin matrices. (3 marks)

   d) Calculate the expectation value of the observable $(\boldsymbol{n}_1 \cdot \underline{\sigma}_1)\otimes(\boldsymbol{n}_2 \cdot \underline{\sigma}_2)$ in the singlet state. (7 marks)

3. a) A system of three qubits $\mathbf{Q}\otimes\mathbf{Q}\otimes\mathbf{Q}$ is to be regarded as partitioned into two subsystems. Is the overall state

$$(|000\rangle + |011\rangle + |100\rangle + |111\rangle)/2$$

separable or entangled? What is the subtlety? Justify your answer. (8 marks)

b) In a system of three qubits, consider the operators $X = \sigma_{1x}\otimes\sigma_{2x}\otimes\sigma_{3x}$ and $O_1 = \sigma_{1x}\otimes\sigma_{2y}\otimes\sigma_{3y}$, $O_2 = \sigma_{1y}\otimes\sigma_{2x}\otimes\sigma_{3y}$, $O_3 = \sigma_{1y}\otimes\sigma_{2y}\otimes\sigma_{3x}$. Show in detail that $O_1O_2O_3 = -X$. (3 marks)

c) State and prove the No-Cloning Theorem. (4 marks)

d) Give a description of the superdense coding protocol. (5 marks)

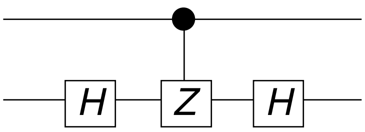
4. a) Write down the usual matrix representation of the CNOT operator. (2 marks)

b) Show that the CNOT gate can be written as

$$\text{CNOT} = |0\rangle\langle 0|\otimes\mathbf{I} + |1\rangle\langle 1|\otimes X$$

(3 marks)

c) Explain carefully what the circuit below does. (5 marks)



d) Give an account of the *Deutch-Jozsa Algorithm.* (8 marks)

e) What would happen in the *Deutch-Jozsa Algorithm* if it was given, as black box, the standard quantum implementation of a boolean function that was not guaranteed to be either constant or balanced? (2 marks)

[PTO]

5. a) If $[a_0 \ldots a_j]$ is a continued fraction, where $a_0 \ldots a_j$ are all integers, show that $[a_0 \ldots a_j] = p_j/q_j$ where p_j and q_j are given by: $p_0 = a_0$, $q_0 = 1$, $p_1 = 1 + a_0 a_1$, $q_1 = a_1$ and for $j \geq 2$,

$$p_j = a_j p_{j-1} + p_{j-2} \quad \text{and} \quad q_j = a_j q_{j-1} + q_{j-2}$$

(10 marks)

b) Write down Shor's algorithm. Comment on the complexities of the various constituent steps. (10 marks)

END OF EXAMINATION