Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Fundamentals of Artificial Intelligence

Date: Monday 22nd May 2017

Time: 09:45 - 11:45

Answer One Question from Section A and One Question from Section B.

Use a SEPARATE answerbook for each SECTION.

This is a CLOSED book examination

The use of electronic calculators is permitted provided they are not programmable and do not store text

Section A

Answer **one** question

1. a) Let a single robot be located within a square arena cluttered with rectangular obstacles and $L_{i,j,t}$ $(0 \le i, j, t \le 99)$ be the event that the robot locates at pose (i, j, t) . Now answer the following questions:

i) Assume that the initial probability that the robot is located at each pose is evenly distributed. Give the formula to calculate the initial probability $P(L_{i,j,t})$?

(2 marks)

ii) Assume that the robot receives a reading o from one of its sensors. Give the formula to calculate $P(L_{i,j,t} \mid o)$

(2 marks)

iii) Assume that the robot takes an action a which is faithfully executed and let $L'_{i,j,t}$ be the event that the robot's new pose (subsequent to the action) is (i, j, t) Give the formula to calculate $P(L'_{i,j,t} \mid a)$

(2 marks)

b) Let a robot be equipped with a sensor which measures the distance to the next obstacle in front of it. The possible distance values between the robot and obstacle are D = 0, 1, 2, …, 9 and the probability of each distance is 0.1. Assume that a sensor reading $o = 5$ is received. It is known that the probability of this reading being accurate is 80%, and the probabilities of being incorrect by +1 or -1 from the real distance are both 10%. Now answer the following questions (Note: you need to explain your reason or give the formulae you use to answer these questions):

i) What is the probability that the distance between the robot and obstacle is less than or equal to 3?

(2 marks)

ii) What is the probability that the distance between the robot and obstacle is 4?

(2 marks)

iii) What is the probability that the distance between the robot and obstacle is 5?

(2 marks)

iv) What is the probability that the distance between the robot and obstacle is more than or equal to 6?

(2 marks)

c) Let a single robot be located within a square arena and facing an obstacle. It is known that the distance between the robot and obstacle is less than or equal to 5 units (that is, the possible distance values are D = 0, 1, 2, 3, 4, 5) with the following probabilities:

$$p(D = i) = 0.1\,(i = 0,1,2,3) \qquad p(D = 4) = 0.2 \qquad p(D = 5) = 0.4$$

Now answer the following questions (Note: you need to explain your reason or give the formulae you use to answer these questions):

i) The robot moves accurately for 2 units unless it encounters the obstacle first it which case it stops. Then what is the probability that the robot's distance to the obstacle is 0 after the action?

(2 marks)

ii) The robot moves 2 units toward the obstacle, but such an action cannot be executed accurately. It is known that there is an 80% chance the action is executed accurately but a 20% chance it moves 1 unit more than the intended action. Now what is the probability that the robot's distance to the obstacle is 2 after the action?

(4 marks)

2. a) State the mathematical definitions of probability distribution and conditional probability. Let A be an event in sample space Ω with its probability $p(A) > 0$ and $p(E \mid A)$ represent the conditional probability of event E under condition A. Now prove that the following function P_A , which assigns a real number in [0,1] to each event E in Ω,

$$P_A(E) = p(E \mid A)$$

forms a probability distribution.

(7 marks)

b) Let E and F be two events and the agent's degree of belief be a probability distribution. Then what is a fair price for the following betting ticket and why?

£x	*if* $E \wedge F$
£0	*if not* $E \wedge F$
Money back	*if not F*

(3 marks)

c) Consider the four-door Monty Hall game in which there is one car behind one of four doors. The rule of playing is as follows: Step 1, you choose a door; Step 2, Monty Hall opens a door with no car behind; Step 3. You switch or stick with your choice; Step 4, Monty Hall opens another door with no car behind. Then Monty Hall will ask you "Do you want to switch or stick with your choice?" Now apply probability reasoning and Bayes' theorem to answer the following questions based on the given sequence of events during the game:

i) Step 1, you choose door 1. Under this event, what is the probability that Monty Hall opens door 2 in Step 2?

(3 marks)

ii) Step 2, Monty Hall opens door 2. Under this event, what is the probability that the car is behind each door?

(2 marks)

iii) Step 3. You switch to door 3. Under this event, what is the probability that Monty Hall opens door 4 in Step 4?

(2 marks)

iv) Step 4, Monty Hall opens door 1. Under this event, what is the probability that the car is behind door 3?

(1 mark)

v) If Monty Hall opens door 4 in Step 4 rather than door 1, is the probability that the car is behind door 3 the same as in the above scenario? Why?

(2 marks)

Section B

Answer **one** question

3. a) The figure below represents a sound waveform. What do the axes of the sound waveform represent? (2 marks)

b) Explain how an audio signal is converted to this format.

(4 marks)

c) How is this data converted into features that are useful for speech recognition?

(4 marks)

d) Given data described by a vector of features $\mathbf{x}$, that can belong to one of two classes, T and F, write an expression that expresses $p(T|\mathbf{x})$ in terms of $p(\mathbf{x}|T)$, $p(\mathbf{x}|F)$, $p(T)$ and $p(F)$. Explain the meaning of each term, how you would generate numerical values for them and how the expression can be used to classify data and give a degree of belief in the classification.

(4 marks)

e) Consider the XOR logic table shown below:

x_1	x_2	XOR
0	0	0
0	1	1
1	0	1
1	1	0

Use the frequency counts from these four examples to estimate $p(x_i|T)$ and $p(x_i|F)$ for $i = 1, 2$, and $p(T)$. Use the naive Bayes assumption to calculate $p(T|x_1 = 1, x_2 = 0)$. How does the classifier perform in this example?

(6 marks)

4. a) Define a first order Markov chain.

(2 marks)

b) A Markov chain model's transition probabilities are shown in the table below.

Some rows are incomplete.

	hh	eh	l	ow	b	ay	END
START	0.5	0.0	0.0	0.0		0.0	0.0
hh	0.5	0.25	0.0	0.0	0.0		0.0
eh	0.0		0.5	0.0	0.0	0.0	0.0
l	0.0	0.0	0.5		0.0	0.0	0.0
ow	0.0	0.0	0.0		0.0	0.0	0.5
b	0.0	0.0	0.0	0.0	0.5		0.0
ay	0.0	0.0	0.0	0.0	0.0	0.5	

i) Complete the table.

(2 marks)

ii) Explain the role of the self-transitions.

(2 marks)

iii) What is the probability of observing the sequence "b-ay-ay"?

(2 marks)

iv) Draw the unrolled version of this model that represents all sequences of length three phonemes.

(4 marks)

v) Using this diagram, calculate the probability that a sequence of three phonemes corresponds to the word "bye".

(5 marks)

vi) Explain how the model could be extended to make a word recognition system. What input is required and how would this be processed to generate suitable input for the model?

(3 marks)

END OF EXAMINATION