COMP 39112

Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Quantum Computing

Date: Wednesday 7th June 2017

Time: 14:00 - 16:00

Please answer any THREE Questions from the FIVE Questions provided

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This is a CLOSED book examination

The use of electronic calculators is NOT permitted

[PTO]

1. a) What does the phrase: "vectors $|x_1\rangle$ ... $|x_n\rangle$ are independent" mean? (2 marks)

   b) What does the phrase "$|x\rangle$ and $|y\rangle$ are orthogonal" mean? (2 marks)

   c) State and prove the Parallelogram Identity. (4 marks)

   d) Define normal, hermitian and unitary matrices. (3 marks)

   e) Write down the three Pauli spin matrices. (2 marks)

   f) What constraints on a and b are required if $a\sigma_x + b\sigma_y$ is hermitian? (1 mark)

   g) What constraints on a and b are required if $a\sigma_x + b\sigma_y$ is unitary? (2 marks)

   h) Let $A = \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix}$. Calculate $\exp(A)$. (4 marks)

2. a) Define a transition system. (4 marks)

   b) Define stochastic and quantum transition systems. What is the key difference between them? (4 marks)

   c) Viewing quantum systems as transition systems, briefly describe the two different kinds of transition that they can undergo. (4 marks)

   d) Assuming the usual notational conventions for eigenvalues and eigenvectors, let the hermitian operator A be written as $|3\rangle 3\langle 3| + |4\rangle 4\langle 4| + |5\rangle 5\langle 5|$. A system is in the state $|3\rangle$. What value can be obtained when A is observed? The system later finds itself in a state $|\psi\rangle$ orthogonal to $|3\rangle$. What value can be obtained when A is observed? (3 marks)

   e) In the system of part (d), let U be the operator $|3\rangle\langle 5| + |4\rangle\langle 3| + |5\rangle\langle 4|$. Show that U is unitary. The system starts in the state $|3\rangle$. What state does it evolve to when U is applied? Suppose that U is then applied again. What state does it evolve to? A is then observed. What value is obtained? (5 marks)

3. a) Explain Basis Copying. Say why it does not violate the No-Cloning Theorem. (6 marks)

b) Derive the Clauser-Horne-Shimony-Holt inequality. Why does quamtum mechanics violate it? (6 marks)

c) Write down the Hadamard matrix. What property does it share with the Pauli spin matrices? (4 marks)

d) In quantum computing, Hadamard operators are extensively used as the first operations applied to the input register. Why? (4 marks)

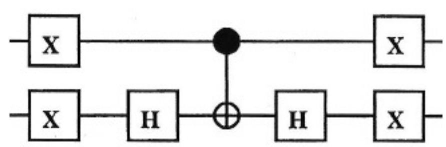
4. a) Write down the usual matrix representation of the C-Θ_ϕ gate, where Θ_ϕ is the phase gate. (2 marks)

b) Which important quantum algorithm does the C-Θ_ϕ gate play a vital role in? (2 marks)

c) Let $V = (1 + \mathrm{i})(\mathbf{I} - \mathrm{i}X)/2$ where X is the NOT gate. Show that $V^2 = X$. (2 marks)

d) Show how with the help of Toffoli gates and a C-U gate, an n-fold controlled C^n-U gate may be implemented. (6 marks)

e) What does the circuit below do? (8 marks)



5. a) Define the quantum Fourier transform, and derive, giving an explanation of the steps, the formula that gives its value for a basis vector. (10 marks)

b) Draw the quantum circuit that implements the quantum Fourier transform and explain how it relates to your answer to part (a). (10 marks)

END OF EXAMINATION