Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Fundamentals of Artificial Intelligence

Date: Friday 18th May 2018

Time: 09:45 - 11:45

Please answer all Questions.

Use a SEPARATE answerbook for each SECTION.

This is a CLOSED book examination

The use of electronic calculators is permitted provided they
are not programmable and do not store text

Section A

1. Let a single robot be located at some position in a square arena with 10x10 = 100 square grids and each possible position is represented as (i, j) (i, j = 0,1,2,…,9). Assume that the initial probabilies that the robot is located at each position are equal, then answer the following questions:

   i) It is known that there is no obstacle within the square arena. What is the initial probability that the robot is located at each position? (1 mark)

   ii) It is known that there are some obstacles within the square arena. Further there is an obstacle detector which provides the following information: isOccupied(i,j) = true if position (i,j) is occupied by an obstacle, and isOccupied(i,j) = false if position (i,j) is not occupied by an obstacle. Give an algorithm or Pseudocode to calculate the initial probability that the robot is located at each position. (5 marks)

2. Let a single robot be located at some position in a square arena with 2x3=6 square grids, in which there are two obstacles occupying positions (0, 2) and (1, 2). The initial probabilities of the robot being located at each unoccupied position are equal. Now the first observation o is obtained from a sensor and the following conditional probabilities are found:

   $$p(o \mid L_{00}) = 1/8 \qquad p(o \mid L_{01}) = 1/4 \qquad p(o \mid L_{10}) = 1/8 \qquad p(o \mid L_{11}) = 1/2$$

   Give the formula to update the probability that the robot is located at position L_{11} under observation o and apply the formula to calculate the value of this conditional probability. (4 marks)

3. Give the mathematical formula of the Extended Total Probability Formula, and then explain how this formula can be used to solve the robot localisation problem when a robot takes an action. (5 marks)

4. Let p be a probability distribute on sample space Ω, and E_0 be an event satisfying $p(E_0) > 0$. Define function $P_{E_0}(E) = p(E \mid E_0)$ which, for every event $E \subset \Omega$, assigns a real number in $[0,1]$. Now use the definition of a probability distribution to prove that P_{E_0} is a probability distribution on Ω. (3 marks)

5. Consider the revised four-door Monty Hall game in which there is one car behind one of four doors. The rule of playing is as follows: Step 1, you choose two doors (say doors 1 and 2); Step 2, Monty Hall opens a door with no car behind (say door 3); Then Monty Hall will ask you "which door is your final choice?" Now apply probability reasoning to answer the question: What is the probability that Monty Hall opens door 3? (2 marks)

Section B

6 Given data described by a vector of features $\boldsymbol{x}$, that can belong to one of two classes, C_i, write an expression that allows you to compute $p(C_1|\mathbf{x})$. Explain the meaning of each term and how their values can be derived. (3 marks)

7 It is useful to use a set of features to characterise data used by a classifier. Consider the problem of recognising spam email.

a What features might be used? (1 mark)

b Describe why the features listed might be useful? (1 mark)

c What data would be used to train the classifier? (1 mark)

d How you would ensure you had a representative sample. (1 mark)

e How critical is it to get the correct prior probabilities? (1 mark)

8 This question makes use of figure 1 that shows the plan of a number of interconnecting rooms. The rooms are used in an experiment to investigate the behaviour of laboratory rats. A rat in a room is equally likely to move to any of the connecting rooms in a single timestep, but cannot remain in a room.

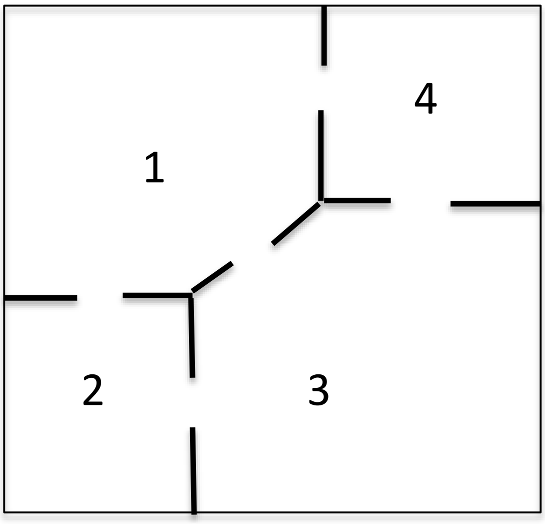

Figure 1

a Write the transition matrix of the probabilities of the rat moving between rooms. (2 marks)

b Now modify the matrix to allow for the rat remaining in a room. (2 marks)

c Calculate the probability of the rat moving from room 1 to room 3 in *exactly* two time steps using the matrix of Part b. (2 marks)

9 This question refers to the Markov model of figure 2.

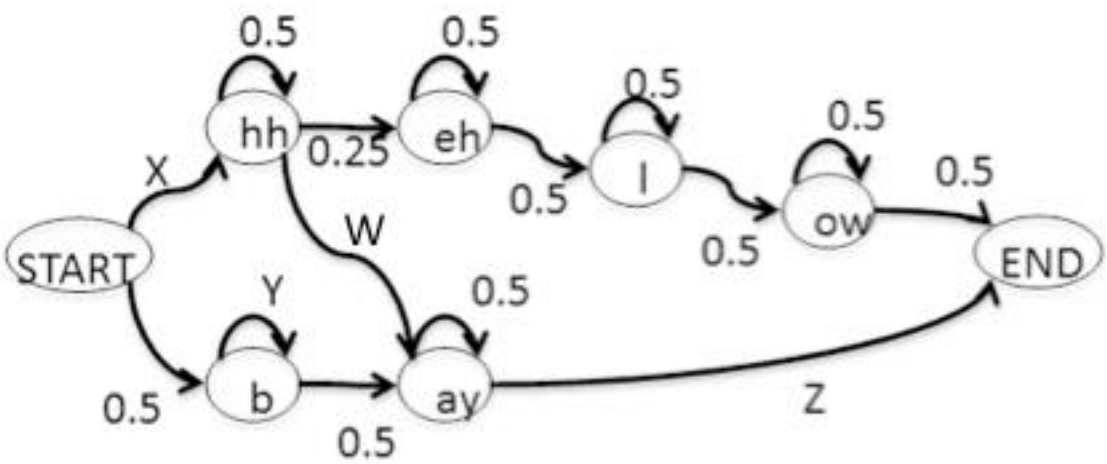

Figure 2

a What words can be generated by the model?

(1 mark)

b How long can the utterances be?

(1 mark)

c Some values are missing, what are they?

(1 mark)

d What is the probability of generating the sequence hh-ay-ay?

(1 mark)

e List the sequences of length 5

(1 mark)

f Compute the probability of a length = 5 sequence being the word “hello”

(1 mark)

END OF EXAMINATION