COMP 39112

Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Quantum Computing

Date: Friday 25th May 2018

Time: 09:45 - 11:45

Please answer all Questions.

This is a CLOSED book examination

The use of electronic calculators is NOT permitted

[PTO]

1. a) What is a basis? (2 marks)

   b) What is an orthonormal basis? (2 marks)

   c) Prove the following Triangle Inequality: $\|x\| - \|y\| \leq \|x+y\|$. (4 marks)

   d) How are the vectors in $\mathbb{C}^n$ characterised? (1 mark)

   e) How are the operators in $\mathbb{C}^n$ characterised? (1 mark)

   f) How are the coefficients λ_k calculated in the expansion $|x\rangle = \sum_k \lambda_k |k\rangle$ of an arbitrary vector $|x\rangle$ when the $|k\rangle$ form an orthonormal basis? (2 marks)

   g) Show that if $\boldsymbol{n}$ is a 3D unit vector, then $(\boldsymbol{n}.\sigma)^2 = \mathbf{I}$. (3 marks)

   h) Show that if $\boldsymbol{n}$ is a 3D unit vector, then $\exp(i\theta\boldsymbol{n}.\sigma) = \cos(\theta)\mathbf{I} + i\sin(\theta)\boldsymbol{n}.\sigma$. (5 marks)

2. a) Write down the axioms of quantum mechanics that define: (i) the state space of a system, (ii) the state space of a composite system versus its components, (iii) the evolution of an isolated system over a period of time. (4 marks)

b) An arbitrary qubit state can be written as $[\cos(\theta/2), e^{i\phi}\sin(\theta/2)]^T$. Derive this. (4 marks)

c) For 1-qubit systems, the operator $\boldsymbol{n}\cdot\underline{\sigma} = (n_x\sigma_x+n_y\sigma_y+n_z\sigma_z)$ is very significant. Explain why. (2 marks)

d) Write down the matrix representation of the operator $\boldsymbol{n}\cdot\underline{\sigma}$. (2 marks)

e) Let $\mathbf{I}$ and X be the identity and NOT operators on a 1-qubit state space. What are the eigenvalues and eigenvectors of $\mathbf{I}\oplus X$? What are the eigenvalues and eigenvectors of $\mathbf{I}\otimes X$? (2 marks)

f) Assume local realistic physics (and, should you need it, counterfactual reasoning). Let A, B, C, be outcomes of classical experiments that can produce results +1 or −1 only. By examining the possible cases, deduce that if ensemble averages are calculated for a large set of trials of these experiments, the inequality $|\langle BA\rangle + \langle BC\rangle| \leq 1 + \langle AC\rangle$ is satisfied. (3 marks)

g) Following on from part (f), now reinterpret the ensemble averages as quantum mechanical expectations (and use as much counterfactual reasoning as you might need). Using the formula $\langle PQ\rangle = -\cos(\angle(P,Q))$, show that the inequality of part (f) can be violated. (3 marks)

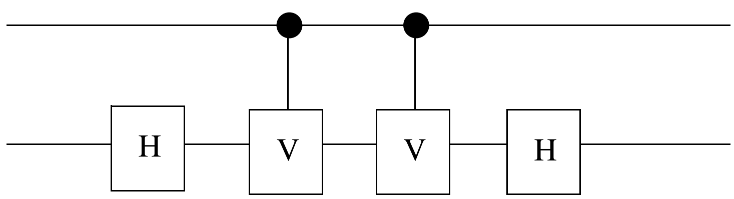
3. a) Write down the matrices for the $\mathbf{I}$, X, Y, Z, Φ_θ operators. (4 marks)

b) How do the four standard basis vectors get mapped by the CNOT gate? (2 marks)

c) Describe what happens to an arbitrary vector in $\mathbf{Q}^{\otimes 2}$ when a measurement is made on the first qubit. (2 marks)

d) Let $V = \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$.

What does the following circuit do? Justify your answer. (4 marks)



e) In the quantum order finding algorithm, write down the unitary transformation U that plays the key role. (4 marks)

f) Referring to part (e), many of the eigenvectors of U depend on the order r, which is the goal. Why is progress possible despite this? (2 marks)

g) What is the elementary algebraic identity that links knowledge of the order r of x modulo N to the factorisation of N? (2 marks)

END OF EXAMINATION