Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Fundamentals of Artificial Intelligence

Date: Friday 17th May 2019

Time: 09:45 - 11:45

Please answer all SIX Questions in Sections A and B, including all sub-questions
Each Section is worth 20 marks

Use a SEPARATE answerbook for each SECTION

This is a CLOSED book examination

The use of electronic calculators is permitted provided they
are not programmable and do not store text

[PTO]

Section A

Use a separate answer book

1. Let a single robot be located within a square arena cluttered with rectangular obstacles and $L_{i,j}$ $(0 \le i, j \le N)$ be the event that the robot is located at position (i, j) . Now answer the following questions:

   i) Assume that the robot receives a reading o from one of its sensors. Give the formula to calculate $P(L_{i,j} \mid o)$ and explain the meaning of each item in the formula.

   (2 marks)

   ii) Assume that the robot takes an accurate action a and let $L'_{i,j}$ be the event that the robot's new position (subsequent to the action) is (i, j). Give the formula to calculate $P(L'_{i,j} \mid a)$ and explain the meaning of each item in the formula.

   (2 marks)

   iii) Assume that $N = 9$, all positions which are not occupied by the obstacles are $L_{i,0}$ $(i = 0,1,...,9)$ (that is, only the first row is unoccupied), and the probabilities that the robot locates in these 10 unoccupied positions is zero except that $p(L_{4,0}) = 0.3$, $(L_{5,0}) = 0.5$, and $p(L_{6,0}) = 0.2$. Now the robot receives a reading $o = 5$ from its left sensor. It is known that this reading is correct with probability 0.8, 1 unit more with probability 0.1, and 1 unit less with probability 0.1. Then what are the probabilities that the robot is located at positions $L_{4,0}$, $L_{5,0}$ and $L_{6,0}$ respectively [that is, calculate $p(L_{4,0} \mid o)$,$(L_{5,0} \mid o)$, and $p(L_{6,0} \mid o)$]. (3 marks)

   iv) Under the same assumptions as given in part iii) above, the robot now moves 5 units toward the left wall. It is assumed that the movement is faithfully executed, except that the robot simply stops when it encounters the wall. Then what are the probabilities that the robot is located at each unoccupied position. (2 marks)

2. Consider the revised five-door Monty Hall game in which there is one car behind five doors. Step 1, you choose two doors; Step 2, Monty Hall chooses a door to open uniformly at random from the doors which do not have a car behind (and is not one of the doors you have currently chosen), and open it; Step 3, you switch or stick with your choice; Step 4, Monty Hall chooses a different door uniformly at random from the remaining doors which do not have a car behind (and is not one of the doors you have currently chosen). Then Monty Hall will ask you "which door is your final choice?" (that is, you can only choose one door at this step, but it can be any door). Assume that a game is played as follows: Step 1, you choose doors 1 and 2; Step 2, Monty Hall opens door 3; Step 3, you switch to doors 4 and 5; Step 4, Monty Hall opens door 1. Now apply probability reasoning and Bayes' theorem to answer the following questions:

   i) After you have chosen doors 1 and 2, based on the knowledge that you have, with what probability would you infer that Monty Hall will open door 3 in Step 2?
   (3 Marks)

   ii) After Step 2, what is the probability that the car is behind each door?
   (2 Marks)

   iii) Which door will be your final choice in order to maximise your chance to choose the door with car behind and why? (3 Marks)

   (Note. For the above questions, one mark for each correct answer and all other marks are for the probability reasoning.)

3. Let p be a probability distribution. For any event A with probability $p(A) > 0$, $p_A(E) = p(E \mid A)$ represents the conditional probability of event E under condition A. Now use the definition of the conditional probability to prove that, for any events E, F, and G with $p(F \wedge G) > 0$,

$$(p_F)_G(E) = p_{F \wedge G}(E) = (p_G)_F(E)$$

(3 marks)

Section B

Use a separate answer book

4. It is useful to use a set of features to describe data.

   What features could be used to describe illnesses?

   How would these features be used to recognise an illness? (2 marks)

5. Multiple measurements can be used to describe the properties of an object, and hence classify the object. A naïve Bayes classifier is often used for this purpose.

   The multiple measurements are recorded as a feature vector $\mathbf{x} = x_i$, and $p(x_i|C)$ is the probability of observing this measurement in a member of class C.

   What is the naïve Bayes assumption?

   How is it used to compute $p(\mathbf{x}|C)$?

   How might be $p(\mathbf{x}|C)$ change if the naïve Bayes assumption is untrue? Justify your answer. (4 marks)

6. Define a first order Markov chain. (2 marks)

A Markov chain model's transition probabilities are shown in the table below. Some rows are incomplete.

	hh	eh	l	ow	b	ay	END
START	0.5	0.0	0.0	0.0		0.0	0.0
hh	0.5	0.25	0.0	0.0	0.0		0.0
eh	0.0		0.5	0.0	0.0	0.0	0.0
l	0.0	0.0	0.5		0.0	0.0	0.0
ow	0.0	0.0	0.0		0.0	0.0	0.5
b	0.0	0.0	0.0	0.0	0.5		0.0
ay	0.0	0.0	0.0	0.0	0.0	0.5	

i) Complete the table. (2 marks)

ii) What is the probability of observing the sequence "b-ay-ay"? (2 marks)

iii) Draw the unrolled version of this model that represents all sequences of length three phonemes. (4 marks)

iv) Using the diagram from part (iii), calculate the probability that a sequence of three phonemes corresponds to the word "bye". (4 marks)

END OF EXAMINATION