Marker’s feedback version

Please answer any THREE Questions from the FOUR questions provided.

Use a SEPARATE answerbook for each SECTION.

For full marks your answers should be concise as well as accurate.

Marks will be awarded for reasoning and method as well as being correct.

The use of electronic calculators is permitted provided they are not programmable and do not store text.
1.

**Marker’s feedback**
This was answered by almost all students, with a high average mark.

a) Explain the term *normal form* as applied to propositional logic formulae, giving descriptions of the three principal normal forms. Describe the procedure to convert a formula into DNF. (6 marks)

**Marker’s feedback**
The most common mistake was to give an answer that was too vague to be described as a procedure. Also many students answered the part about normal forms without saying what characterises the 3 principal normal forms.

b) Let \( A \) be the propositional logic formula
\[
(((p \lor q) \Rightarrow r) \lor ((p \lor r) \Rightarrow q)) \Rightarrow (r \Rightarrow q)
\]

i) Find a formula in CNF form that is logically equivalent to \( A \). (6 marks)
ii) Is \( A \) a tautology? Justify your answer. (2 marks)

**Marker’s feedback**
Some students made it very difficult for themselves by failing to remember that \( a \Rightarrow b \) is \( \neg a \lor b \); this is one you MUST learn.
Common mistakes were

- mis-parsing the expression: it was of the form \( (A \lor B) \Rightarrow C \), but some people read it as \( A \lor (B \Rightarrow C) \).
- Forgetting brackets. Expressions such as \( (p \lor q) \land \neg r \) were often written without brackets and then interpreted as \( p \lor (q \land \neg r) \).
- Failing to use absorption to simplify. The almost final expression is of the form
\[
(X \land Y \land Z \land \neg r) \lor \neg r \lor q
\]
which simplifies to \( \neg r \lor q \).

c) The predicates \( S \) and \( L \) are defined by

\[
S(x) \iff x \text{ is a student}
\]
\[
L(x) \iff x \text{ is allowed to use the library}
\]

Express in first order logic the statements
i) All students are allowed to use the library (2 marks)
ii) Only students are allowed to use the library (2 marks)
iii) Some non-students are allowed to use the library (2 marks)

**Marker’s feedback**
Generally answered well, although many people gave the unlikely answer $\exists x \ -S(x) \Rightarrow L(x)$, for the last part.
2. **Marker’s feedback**
This was answered by almost all students.

a) Explain what is meant by the term *formal language*, illustrating your answer with examples. (4 marks)

b) A small formal language $R$ of *expressions* is defined by the rules:

- The lower case letters $a, b, \ldots, z$ are *atoms*.
- Any atom is a *term*.
- If $T$ is a term and $A$ is an atom, then $T \ast A$ is a term.
- Any term is an expression.
- If $E$ is an expression and $T$ is a term, then $E + T$ is an expression.

i) Give three examples of expressions in this language, giving the parse tree for each. (3 marks)

**Marker’s feedback**
No great problems here except that some students gave expressions which included upper case letters and brackets, which do not occur in the language.

ii) Show that the following is an expression in $R$ by giving its parse tree:

$x \ast y + w + x \ast y \ast z + y \ast x$

(8 marks)

**Marker’s feedback**
The most common error here was not to recognise that the right hand argument of a $+$ must be a Term, not a general expression.

c) The binary operation $\ast$ is defined on the set $\mathbb{Q}$ of rational numbers by

$$p \ast q = p + q + 2pq$$

i) Determine whether $\ast$ is commutative. (2 marks)

ii) Determine whether $\ast$ is associative. (3 marks)

Justify your answers.
Marker’s feedback
The most common, and annoying, errors were the following

- Attempting to prove equality if two expressions by evaluating both for a small number (often 1) of examples. No matter how many values you substitute, this does not constitute a proof of equality of expressions in general.

- 0 = 0 proofs. You should not attempt to prove the equality of two expressions by assuming they are equal and then deriving something like 0 = 0. You need to show that the left hand side is equal to the right.