COMP11120
Corrigenda for Semester 2

Example 6.18, p287. The calculation that follows the definition uses the wrong base case for the hght function. The calculation should read:

Example 6.18.

$$
\begin{aligned}
&\mathrm{hght}\,\texttt{tree}_5(\texttt{tree}\,3, \texttt{tree}_4(\texttt{tree}\,2, \texttt{tree}\,1)) & \\
&= \max\{\mathrm{hght}\,\texttt{tree}\,3, \mathrm{hght}\,\texttt{tree}_4(\texttt{tree}\,2, \texttt{tree}\,1)\} + 1 & \text{step case hght} \\
&= \max\{0, \max\{\mathrm{hght}\,\texttt{tree}\,2, \mathrm{hght}\,\texttt{tree}\,1\} + 1\} + 1 & \text{def hght} \\
&= \max\{0, \max\{0, 0\} + 1\} + 1 & \text{base case hght} \\
&= \max\{0, 1\} + 1 & \text{def max} \\
&= 1 + 1 & \text{def max} \\
&= 2 &
\end{aligned}
$$

Example 7.59, p394. The recursively defined relation gives the identity relation, rather than the relation described in the text. In order to fix this the recursive definition has to be adjusted, which requires changes to various proofs made for the relation. I include the complete corrected example below.

Example 7.59. We show how to define a partial order on $\mathsf{FBTrees}_S$, where S is an arbitrary set. We would like to define[1]

Base cases $\leq$. For all $s \in S$, we have $\texttt{tree}\,s \leq \texttt{tree}\,s$.

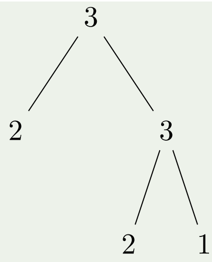
For all $s \in S$, t, $t' \in \mathsf{FBTrees}_S$ we have $\texttt{tree}\,s \leq \texttt{tree}_s(t, t')$.

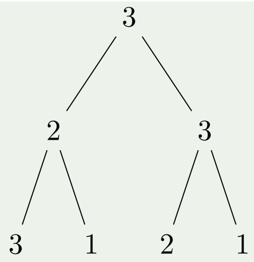
Step case $\leq$. If $t \leq t''$ and $t' \leq t'''$ then $\texttt{tree}_s(t, t') \leq \texttt{tree}_s(t'', t'')$.

This relates two trees if and only if the second arises from the first by repeatedly extending the tree, that is

- pick a leaf with label s (that is a subtree of the form $\texttt{tree}\,s$) and
- replace that leaf by a tree whose root label is s, that is, a tree of the form $\texttt{tree}_s(t, t')$.

So, for example, we have that the tree on the left is less than or equal to the one on the right.





Alternatively we can think of the relation $\leq$ as relating a tree t to a tree t' if and only if we can obtain the first tree from the second tree by cutting off some of the branches.

We can show that this is a partial order. For reflexivity we have an inductive proof.

Base cases $\leq$. For $s \in S$ we know that $\texttt{tree}\, s \leq \texttt{tree}\, s$ by the first base case of $\leq$.

Ind hyp. We know that that $t \leq t$ and $t' \leq t'$ for some full binary trees t and t' over S.

Step case $\leq$. From the induction hypothesis we may use the step case of $\leq$ to deduce that for every $s \in S$ we have

$$\texttt{tree}_s(t, t') \leq \texttt{tree}_s(t, t').$$

We can show anti-symmetry by a second induction proof.

Base cases $\leq$. For $t, t' \in \mathsf{FBTrees}_S$ if $t \leq t'$ by one of the base cases of the definition then it must be the case that we can find $s \in S$ such that $t = \texttt{tree}\, s$.

In order for $t' \leq t$ to also hold it must be the case, since we know that $t = \texttt{tree}\, s$, that this also occurs by the first base case of the definition of $\leq$, and so we must have that there is $s' \in S$ with $t' = \texttt{tree}\, s'$, but But if they are related by the base case of the definition of $\leq$ then it must also be the case that $s = s'$, and so

$$t = \texttt{tree}\, s = \texttt{tree}\, s' = t'.$$

Ind hyp. If $t \leq t'$ and $t' \leq t$ then $t = t'$.

Step case $\leq$. The only remaining case is that

$$\texttt{tree}_s(t, t') \leq \texttt{tree}_s(t', t'') \qquad \text{and} \qquad \texttt{tree}_s(t', t''') \leq \texttt{tree}_s(t, t')$$

both by the step case, and so

$$t \leq t' \text{ and } t'' \leq t''' \qquad \text{and} \qquad t' \leq t \text{ and } t''' \leq t''$$

and by the induction hypothesis

$$t = t' \qquad \text{and} \qquad t'' = t'''.$$

The proof of transitivity is a similar induction proof, see the following exercise.

Exercise 166, p395. Please replace part (c) of this exercise by the following statement:

(c) Show that we have for $t, t' \in \mathsf{FBTrees}_S$ and the partial order $\leq$ from Example 7.59 that

$$t \leq t' \qquad \text{if and only if} \qquad (t, t') \in R_{\leq}.$$