First some general remarks. The exam makes up 75% of the final mark for this unit with the remaining 25% coming from the coursework mark.

193 students sat the exam. The median mark was 32 out of 60 (or 60%), and the average 30.2. Fifty-one student had a failing mark. However, a significant proportion of these are close to the border, and if they have good coursework marks they should pass the unit. Seventeen student had a mark below 15 (which is below 30%), with a lowest mark of 0 (out of 60). On the top end, forty-four students achieved first class marks with a top mark of 56, which is 93%.

Looking at the distribution among questions, 193 students answered Question 1, 186 Question 2, 138 Question 3 and 52 Question 4. Since Question 4 had a higher average mark than Question 3 (and a higher average than Question 1) one has to wonder whether students could accurately judge their knowledge.

Statistical analysis of individual questions:

**Question 1.** This had an average mark of 10.5 (52.5%), with 46 students on a failing and 56 students on a first class mark.

**Question 2.** This had an average mark of 12.3 (61.5%), with 20 students on a failing and 76 students on a first class mark.

**Question 3.** This had an average of 7.9 (39.4%), with 66 students on a failing, and 26 on a first class mark.

**Question 4.** This had an average of 10.5 (52.7%), with 21 students on a failing, and 23 students on a first class mark.

Here is an overview of the percentage distribution if we split the marks into degree classes.

<table>
<thead>
<tr>
<th></th>
<th>Fail</th>
<th>3rd</th>
<th>low 2nd</th>
<th>up 2nd</th>
<th>1st</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Question 1</td>
<td>24%</td>
<td>17%</td>
<td>19%</td>
<td>11%</td>
<td>29%</td>
<td>Marks (out of 20)</td>
</tr>
<tr>
<td>Question 2</td>
<td>11%</td>
<td>13%</td>
<td>16%</td>
<td>20%</td>
<td>39%</td>
<td></td>
</tr>
<tr>
<td>Question 3</td>
<td>47%</td>
<td>12%</td>
<td>13%</td>
<td>9%</td>
<td>19%</td>
<td></td>
</tr>
<tr>
<td>Question 4</td>
<td>40%</td>
<td>6%</td>
<td>4%</td>
<td>6%</td>
<td>44%</td>
<td></td>
</tr>
<tr>
<td><strong>Overall</strong></td>
<td>26%</td>
<td>16%</td>
<td>22%</td>
<td>13%</td>
<td>23%</td>
<td>Marks (out of 60)</td>
</tr>
</tbody>
</table>

Comments on exam technique: I'm not sure students necessarily made a good choice regarding which question to answer. I saw very few students use sample words to check the correctness of their answer. Also, where I asked for different descriptions of the same language it would have been prudent to check whether the given solutions agreed, and I couldn’t see much sign that students were doing this.

**Questions from Section A.**

More detailed comments on answers to questions from Section A.

**Question 1.** Overall, there were quite a few small mistakes in each part of (a) for many students. For part (b), being able to use Algorithm 2 proved a significant hurdle for quite a few. The main reason the average for this question is
not higher is the number of students on marks from 6 to 8 (37). The highest mark achieved for this question was 18, the lowest 0 (each by three students).

a) Part (i) was very well answered overall. Mistakes that did occur were a misunderstanding of ‘last but one’ (although this did appear in marked exercises and should therefore have been known), and students not realizing that all two letter words (aa, ab, ba, bb) are elements of the language. Where this mistake was made repeatedly marks were only deducted once. Somewhat to my puzzlement some answered this question as if the underlying alphabet had been \{a, b, c\}. Clearly the most difficult part of this question was (ii), which is why it was worth four marks. Every student realized that the desired automaton would have two symmetric parts. The smallest automaton which works correctly has 9 states (one start state and four states in each of the two parts). Checking whether sample words would have been a good idea. Quite a few students started with an NFA, but that had to be turned into a DFA for full marks (an NFA is much easier to give). Part (iii) was well answered too, and typically mistakes occurring seemed to be due to students looking for a short answer. Part (iv) was answered correctly by few students as expected. Many attempts did not stick with the required format.

b) Giving the highest number, 2, to the start state is by far the most convenient numbering of the states, but very few students did that. Note that in this case 

\[ L_{\leq 2}^{a} = (L_{\leq 1}^{a})^{\star} \]

which makes the model answer quite short. Typically students either did not recall the algorithm correctly or they made multiple mistakes when reading off languages. Very few students achieved full marks here, and typically those who did not use Algorithm 2 managed at best half marks. The second part was reasonably well answered. Those who reasoned that they had already found the shortest form for their regular expression were awarded full marks for this part.

**Question 2.** By and large the quality of answers for this question was good. The average was really dragged down by the students who seemed to be completely unprepared (14 students had a mark of 5 or less). Eight students received full marks for this question (the lowest mark was 0).

a) The vast majority of students could give a word that has two parse trees (the shortest such is aaaa). Some last marks by not giving parse trees (giving derivations is not sufficient), but almost everybody got full marks for part (i). Somewhat to my surprise some students were confused about the meaning of ‘ambiguous’ and treated it as if it meant unambiguous. Again, part (ii) had almost everybody get full marks with the exception of the few who seemed puzzled by what the question might mean. I was really pleased by the fact that the majority of students also answered part (iii) correctly (all words over the given alphabet which start with a and which contain at least two instances of the letter a). Regular expressions for the same language \((a(a|b|c)^{\star}a(a|b|c)^{\star})\) also received full marks. Part (iv) was a bit more hit and miss, but was still answered correctly by many. The trick is to force the second a in each word to be either the second, or the last, occurrence of that letter. A handful of students gave a DFA and turned that into a grammar.

b) This was well answered, by and large. Mistakes that occurred when applying Algorithm 1 typically were due to students not keeping track carefully enough about which states could be reached with which letters, or by not
creating outgoing transitions from states already created. Students who did not use the algorithm typically made a poor job of giving an answer.

c) I had originally intended to give two automata where one direction is possible, and the other isn’t, but I left it as it was when I created something slightly different. Neither direction allows a simulation. From left to right one is forced to keep adding pairs to the simulation and eventually an accepting state on the left has to be matched to a non-accepting state on the right, which is not allowed. From right to left one has a choice regarding what state to match to 1. But one of the candidates, B, requires the pairing 2 with B, which again means pairing an accepting state with a non-accepting state, and the other candidate, D, has no matching transition labelled c, so this also fails. For each direction I gave one mark for correctly stating whether a simulation exists, one mark for beginning to build a simulation correctly, and one mark for giving a correct reason of the simulation not being completable.