Exercise Sheet 8 for examples class in Week 9

1. For each of the following specifications, give a program \( S \) that satisfies the specification, or if there is no such \( S \) explain why.
   (a) \( \{ \} S \{ x = y \} \)
   (b) \( \{ x = a \} S \{ z = 2 \times a \} \) where \( a \) is not in \( S \)
   (c) \( \{ x = a \land y = b \} S \{ a < z < b \} \) where \( a, b \) is not in \( S \)
   (d) \( \{ false \} S \{ true \} \)
   (e) \( \{ true \} S \{ false \} \)
   (f) \( [ true ] S [ false ] \)
   (g) \( \{ x = a \land y = b \land x > y \} S \{ y > x \} \) but not \( \{ x = a \land y = b \land x > y \} S [ y > x ] \)

2. Give a proof for the following partial correctness problem
   \[ \{ x = a \land y = b \land x \geq 0 \} \text{while} x > 0 \text{ do } (x := x - 1; y := y + 2) \{ y = b + (2 \times a) \} \]
   You will need to find a loop invariant for the loop first.

3. Give a proof for the following total correctness problem
   \[ [ x = a \land y = b \land x \geq 0 \land 2x < y ] \text{while} x > 0 \text{ do } (x := x - 1; y := y - 2) [ y > 0 ] \]
   You will need to find both a loop invariant and a loop variant for the loop first.

4. Extend the partial correctness proof of the GCD algorithm given in 2.3.3 to show total correctness e.g.
   \[ [ x > 0 \land y > 0 ] C [ z = \text{gcd}(x, y) ] \]
   You will need to find a loop variant for the loop.

5. Exercise 2.12 i.e. prove total correctness for the logarithm program you wrote in Exercise 1.11 with respect to a total version of the specification you wrote in Exercise 2.3