ENHANCING A DECISION SUPPORT TOOL WITH SENSITIVITY ANALYSIS

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List of Acronyms

AHP	Analytic Hierarchy Process
CI	Consistency Index
CR	Consistency Ratio
DM	Decision Maker
EV	Eigenvector
GM	Geometric Mean
GDM	Group Decision Making
IAHP	Interval AHP
J2ME	Java 2 Micro Edition
JIT	Just In Time Compiler
JVM	Java Virtual Machine
MCDM	Multi-Criteria Decision Making
MIDlet	MIDP Application
MIDP	Mobile Information Device Profile
MVC	Model View Controller
NV	Number of Violations
OAT	One At a Time
OPSC	Operating Point Sensitivity Coefficient
PC	Pairwise Comparison
PrInT	Prioritisation using Indirect Judgements
RR%	Rank Reversal Probability
SA	Sensitivity Analysis
SC	Sensitivity Coefficient
TD	Total Deviation
TD2	Total Deviation Using Indirect Judgements
TDD	Test Driven Development
TSC	Total Sensitivity Coefficient
WSM	Weighted Sum Model

Abstract

With the increased speed and complexity of today's world, and the ever increasing amount of data available, a decision support system is of critical importance.

Multi-Criteria Decision Making (MCDM) enables decision makers to make a decision considering several alternatives and multiple criteria of evaluation. The Analytic Hierarchy Process (AHP) is an area of research in MCDM where the problem is decomposed into a hierarchical model in order to rank the alternatives. Sensitivity analysis (SA) is a technique that determines the effects of changes in input values on a model results, hence, performing an SA on the results of a decision problem may provide valuable information to the decision maker about the robustness of the solution.

This work investigates various methods to carry out SA on AHP models. In addition, a framework for evaluating software in terms of AHP features and SA support is developed and a number of commercial and academic software tools are analysed using this framework. The result from this analysis shows that despite the existence of several approaches to performing SA, available software tools only provide a basic form of SA. As a consequence, based on the framework analysis, an appropriate tool is enhanced to include an SA module.

In summary, the implementation of additional features and improvements involves a re-engineering process of the AHP tool. Because of the re-engineering activity, the performance of the re-designed tool has been improved by a factor of two compared to the original version. Additionally, as a consequence of this re-engineering, the enhanced tool is also made available on web and mobile platforms. Further, a major challenge in the area is the problem of selecting a single solution from a set of non-dominated solutions generated from the input judgements; to make progress towards this, a selection method is developed using one of the implemented SA algorithms.

Declaration

No portion of the work referred to in this dissertation has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.

Intellectual Property Statement

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Chapter 1
Introduction

1.1 Project Context

Everyone has to make decisions, all the time. Some decisions are very simple and have such a small impact that they can be taken without much analysis. In contrast, there are complex decisions with a significant impact that have to be taken with as much information, analysis and reflection as possible to decide on the possible alternatives. Managerial decisions may have a significant impact on the performance and success of companies. For example, Campbell et al. (Campbell, 2009) and Janis (Janis, 1972) have identified examples where clearly very wrong decisions have been made. With the increased speed and complexity of today's world and the ever increasing amount of data available, decision support is of critical importance.

Decision making is a structured process that formalises the steps involved in making decisions. *Multi-Criteria Decision Making* (MCDM) is a discipline that enables decision makers to make a decision considering several alternatives and multiple criteria of evaluation. MCDM has become so attractive that several books, thousands of articles and many scientific journals are dedicated to the subject (Figeroa, 2005).

The *Analytic Hierarchy Process* (AHP) (Saaty, 2008) is an area of research in MCDM where the problem is decomposed into a hierarchical model, represented as a tree. The top element of the hierarchy represents the overall goal, intermediate nodes correspond to the different criteria and the leaves of the hierarchy tree are the alternatives. The relative importance of the alternatives and criteria is assessed by using the Pairwise Comparison (PC) method. Once all nodes in the hierarchy tree are evaluated, then the aggregated importance of the alternatives is calculated and a solution or ranking is obtained, known as the *preference vector*.

To assess the importance of alternatives and criteria using PC, only two elements are compared at a time. The value assigned to this comparison, or judgement, may have an objective or subjective origin. In either case, the judgements represent the direction of preference between the two elements and the strength of this relation. For each criterion a matrix is formed with the judgements and an elicitation method is used to obtain the preference vector for this matrix containing the weights of each element.

Often, decision makers (DM) are not completely confident about their judgements they assign to each comparison; the judgements may be too subjective or may come from a group decision where members may have different opinions about each criterion (Erkut, 1991). In such cases, it is desirable to run a sensitivity analysis (SA) on the results to analyse how sensitive the solution is to changes in input data. SA is a technique that determines the effects of changes in input values on a model's results. In this way, SA is a powerful tool to assess how the preference vector is affected by changes in the PC judgements or in the weights of the elements. SA may be useful in providing information equally important to the solution of the problem (Chen, 2008) such as analysis of the robustness of the recommended solution, identification of critical elements in the model and help to answer “what if” questions.

Several software tools provide support for both AHP and some level of SA; some have been in the market for many years, e.g. Web-HIPRE (Mustajoki, 2000), while others have been developed recently, e.g. MakeItRational (MakeItRational, 2012). Given the importance and popularity of the AHP field and the usefulness of SA, various approaches in the literature for performing SA will be analysed. Based on these analyses, a framework of desirable features for AHP software will be constructed and various software tools will be assessed according to this framework. The objective of this framework is to assess the potential of AHP software to deliver a set of important features necessary to provide a satisfactory solution to a decision problem, and identify potential improvements that may help the DM to make more robust decisions.

The framework has identified *PriEsT* (Priority Estimation Tool), an MCDM AHP

tool developed at the University of Manchester ((Siraj, 2011), (Siraj, 2012a)) available for Linux and Windows, as the best current offering despite it not offering support for SA. To implement additional SA features in PriEsT, a process of re-engineering is important to analyse the application and to improve its modularity, code readability and reduce the time needed to add new functionalities (Sommerville, 2000). As a consequence of this re-engineering, the enhanced PriEsT tool will also be made available on web and mobile platforms.

1.2 Aims and Objectives

The aims and objectives of this project were to:

1. Investigate how to perform sensitivity analysis in the context of AHP and the different methods that are available in the literature.
2. Perform a survey of the software tools that support AHP and SA, and develop a framework that enables comparison in terms of desirable features or requirements.
3. Select an appropriate MCDM tool for development – PriEsT has been selected.
4. Improve PriEsT with support for additional desirable features (SA, and web and mobile deployment).

The general aim of this research and implementation project was to investigate methods to perform SA in AHP. Following this, PriEsT was analysed, re-engineered as appropriate, and enhanced by including a module for SA. As a consequence of this re-engineering various further enhancements were identified and developed, such as additional platform availability.

To accomplish the goal of the project the following objectives were defined:

- Explore AHP and SA methods. Review the literature to identify the existing methods to perform SA.
- Re-engineer and re-structure PriEsT in a modular way. Create a library

with the AHP functionalities and SA.

- Following this re-engineering, develop further enhancements that will provide additional platform availability:
  - a web version of PriEsT as a Java Applet.
  - a mobile version of PriEsT as a J2ME MIDlet[1].
- Carry out an evaluation to measure the performance and efficacy of SA methods and performance of platform-specific versions of PriEsT.

1.3 Contributions

1.3.1 Implementation of three SA algorithms

Three SA algorithms are studied and implemented: one-at-a-time, probabilistic simulations and mathematical modelling. Implementation details are given for each algorithm; in addition, performance tests are executed and the methods are compared to evaluate their efficacy in analysing decision problems. Based on our literature analysis and evaluation framework, the enhanced PriEsT tool would appear to be the only one that has three approaches to SA.

1.3.2 Re-engineering and enhancing PriEsT with SA.

The PriEsT tool is re-engineered and, as a consequence, an open source Java library is developed, named the PriEsT library. This library contains a set of functionalities to work with AHP problems and SA.

In addition, a new platform-independent version of PriEsT is designed and implemented using Java. This application uses the PriEsT library as the core and allows users to work with decision problems and perform the three different types of SA.

Because of the re-engineering activity, the performance of the re-designed tool

1 A MIDlet is an application written in Java Micro Edition targeted to run on mobile devices using the MIDP profile.

has also been improved by a factor of two compared to the original version.

1.3.3 Method for selecting a single solution for a PC matrix

A major challenge in the AHP and PC area is the problem of selecting a single solution from a set of non-dominated solutions generated from the input judgements; to make progress towards this, a novel selection method has been developed to evaluate and select a single solution using the mathematical modelling algorithm for performing SA.

1.3.4 Implementation of a web version of PriEsT

A web version of PriEsT is implemented as a Java Applet. This web application is built on top of the desktop application and the PriEsT library and allows PriEsT to run on any web-browser that supports Java without the need to download or install any software.

1.3.5 Implementation of a mobile version of PriEsT

Two mobile versions of PriEsT are developed: one for the J2ME platform and one for the Android platform. These applications are compatible with the desktop and web version, so problems files created with any version can be interchanged among applications.

1.4 Dissertation Structure

This dissertation contains eight chapters. The present chapter has introduced the project's context, aims and objectives and has presented the contributions of the work.

Chapter 2 reviews the multi-criteria decision making field. Relevant material about the Analytical Hierarchy Process (AHP) and pairwise comparison (PC) is given. The chapter ends with an examination of the techniques to perform sensitivity analysis (SA) in AHP models.

Chapter 3 presents a survey of several decision support tools and performs a

comparison to evaluate which tools support the features presented in Chapter 2. As the PriEsT tool obtained the highest score in the evaluation, it is selected to be enhanced by including the SA techniques identified in Chapter 2.

In order to enhance PriEsT, a re-engineering process is carried out as discussed in Chapter 4. The motivations for the re-engineering process are laid out; an analysis of the architecture and source code is presented; new requirements are introduced; finally, a new design, implementation methodology and tools are presented.

Chapter 5 discusses the design and implementation of three different SA methods. Details about the implementation along with the main advantages and limitations of each method are presented. The chapter concludes with the presentation of a case study to demonstrate the use of the SA methods.

The advantages of the re-engineering process allows the creation of a web and mobile version of the application, which are presented in Chapter 6. Architecture design and implementation details are covered. In addition, the challenges faced during the implementation process are considered.

Chapter 7 presents the evaluations and system testing performed in the project. A traceability matrix is presented to evaluate the fulfilment of requirements and performance tests are discussed for the desktop and mobile applications. The chapter concludes by presenting the results of specific performance tests to evaluate the SA algorithms.

Chapter 8 concludes the project. A summary of the project and its main achievements are presented, followed by several suggestions for future work. A brief discussion is presented reviewing the project plan and how each objective was achieved. Finally, the dissertation is finalised with the presentation of concluding remarks.

Supplementary materials that are referenced in the dissertation are given in the appendices.

Chapter 2
Project Background

This chapter presents relevant background material. First, an introduction to the Multi-Criteria Decision Making field is given. Then, the Analytic Hierarchy Process and the Pairwise Comparison methods are described. Next, different techniques for performing sensitivity analysis are discussed. Finally, desirable features of AHP software and existing tools are considered.

2.1 Multi-Criteria Decision Making

Multi-Criteria Decision Making (MCDM) is an area of Operational Research that supports the process of decision making. In MCDM, the goal is to rank different alternatives considering multiple, often conflicting, criteria. For example, consider buying a new car, some of the criteria to consider and evaluate are cost, fuel consumption, safety, capacity and style. After evaluating a list of possible cars against these criteria, a ranking of cars can be obtained and the most appropriate choice can be selected.

For an alternative to be judged by a criterion, a scale of possible values must be defined for the criterion. A scale is defined by the direction (cost or benefit) and the magnitude of the values. In addition, different types of criteria may be used including measurable, ordinal, probabilistic or fuzzy criteria (Jacquet-Lagrèze, 2001).

To find the highest scoring alternative, the DM must evaluate all possible choices against each criterion. Then a prioritisation method is applied to aggregate all judgements and create a ranking of the alternatives. Finally, the DM uses this information as a recommendation to select one of the alternatives according to his/her requirements and preferences.

The MCDM methodology is a process that includes four main steps beginning with the definition of the decision problem and ending with the selection of an alternative (Jacquet-Lagrèze, 2001). The steps are as follows:

1. **Structuring the decision problem:** the DM establishes the problem and the set of possible alternatives to consider.
2. **Modelling the criteria:** the criteria and the way to measure the alternatives for each criterion are defined.
3. **Aggregating the preferences:** each alternative is judged against each criterion and an aggregation method is used to derive the ranking of alternatives.
4. **Recommendations:** recommendations are given to the DM based on the results from the previous step. The DM selects one of the alternatives.

There exist several prioritisation methods[2] (Triantaphyllou, 2000) that aggregate the preferences in step 3 and different methods may yield different results. Although studies (Guitoni, 1998) have compared different methods and introduced frameworks for selecting the most appropriate depending on the problem, according to a study from Wallenius et al. (Wallenius, 2008) in 2008, the most popular method in the literature since the 1970s has been the Analytical Hierarchy Process (AHP). The AHP method is described in the next section.

2.2 Analytic Hierarchy Process

AHP is a method of prioritisation that enable DMs to evaluate the relative importance of objective and subjective alternatives and criteria by using the pairwise comparison technique. It was introduced by Saaty in 1980 (Saaty, 1980) and has proven very popular worldwide since its creation (Wallenius, 2008).

To apply the AHP method a decision problem must be decomposed into four steps (Saaty, 2008):

2 The most popular methods are: weighted sum model (WSM), weighted product model (WPM), analytic hierarchy process (AHP) (Saaty, 1980), ELECTRE (Roy, 1968), TOPSIS (Hwang, 1981), SMART (Edwards, 1977), PROMETHEE (Brans, 1984), multi attribute utility theory (MAUT) (Wallenius, 2008), UTA (UTilités Additives) (Jacquet-Lagrèze, 1982)

1. Define the problem and determine the outcome sought
2. Structure the problem as a hierarchy, where the top element is the goal of the decision. The intermediate levels define the criteria on which the set of alternatives in the lowest level will be judged.
3. Construct one PC matrix for every non-leaf node in the hierarchy and get the priority vector from each matrix. Each element in an upper level is used to evaluate its child elements with respect to it.
4. Aggregate the priorities obtained from the PC matrices. The priorities in one node are used to weight the priorities in the level below and then are added to obtain the global priority. This method is known as the Weighted Sum Model (WSM). The alternative with the highest global priority is considered to be the best choice.

For instance, consider a decision problem with M alternatives A_i (for $i=1...M$) and N criteria C_j (for $j=1...N$). Let w_j be the weight of criterion C_j and $\delta_{i,j}$ be the performance value of alternative A_i for criterion C_j and P_i be the overall priority of alternative A_i.

AHP use normalised weights, that is, the sum of weights must be 1, as shown in formula (1).

$$\sum_{i=1}^{M} \delta_{i,j}=1, \qquad \sum_{j=1}^{N} w_j=1, \qquad \sum_{i=1}^{M} P_i=1 \tag{1}$$

The overall priorities P_i of the alternatives are calculated using the WSM as shown in formula (2):

$$P_i=\sum_{j}^{N} \delta_{i,j} w_j \quad , for\ i=1,...,M \tag{2}$$

For simplicity only one level of criteria is presented. For more than one level, formula (2) is applied at every level of the hierarchy. For example, in Figure 2.1 a hierarchy with two levels of criteria and three alternatives is presented. To obtain the overall priority for alternative A_1, firstly the local priorities for

criterion C_1 and C_2 have to be calculated. Formula (2) is then applied in node C_1 using the weights of C_{11} and C_{12} and the performance value of A_1 for these two criteria to get the local priority in criterion C_1:

$$P_{1,1}=\delta_{1,11}w_{11}+\delta_{1,12}w_{12} \tag{3}$$

For criterion C_2 the local priority $P_{1,2}$ is the same as $\delta_{1,2}$ as there is no intermediate nodes. Next, formula (2) is again applied to calculate the global priority:

$$P_1=P_{1,1}w_1+P_{1,2}w_2 \tag{4}$$

Using this method any hierarchy, independent from the number of levels, can be analysed and global priorities can be obtained.

To assess the alternatives against each criterion a scale of dimensionless numbers is used. That is, even though the criterion may be referring to cost or time, when the PC method is used the units of measure become abstract units and in this way the aggregation is possible for different scales.

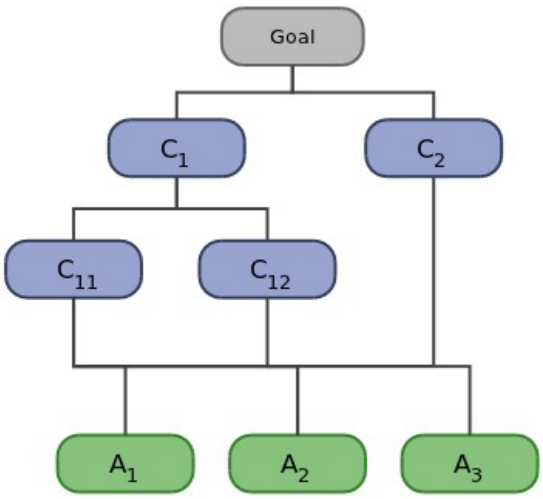


Figure 2.1. A hierarchy model in AHP

Belton and Gear (Benton, 1983) found that when adding an alternative that is identical or similar to an existing one, the ranking of options may be reversed.

To prevent this, they developed the *Ideal Mode AHP*. In this mode, instead of having the performance values of the alternatives for a given criterion sum to 1, each value is divided by the maximum value in the vector so the resulting vector has a maximum value of 1 instead of the sum being 1.

2.3 Pairwise Comparison (PC) Method

To assess the alternatives under a given criterion, it is often very hard for DMs to assign an absolute score. Qualitative and quantitative data may be unavailable or necessary information to quantify the performance of alternatives may be incomplete. Therefore, the PC method is used to determine the relative importance or weights of the alternatives and criteria with respect to each criterion in the decision problem.

Under this approach, the DM has to analyse only two elements at a time. To make this comparison, the DM has to choose a value indicating how many times more important, preferred or dominant one element is over another element in terms of a given criterion. This value has to be given in reference to a predefined scale. Saaty (Saaty, 1980) proposed a ratio scale from 1 to 9 (see Table 2.1). Other scales have been proposed; Triantaphyllou et al. (Triantaphyllou, 1994) evaluated 78 different scales and concluded that there is no scale that outperforms all other scales.

Intensity of Importance	*Definition*	*Explanation*
1	Equal Importance	Two activities contribute equally to the objective
3	Moderate importance	Experience and judgement slightly favour one activity over another
5	Strong importance	Experience and judgement strongly favour one activity over another
7	Very strong importance	An activity is favoured very strongly over another; its dominance demonstrated in

Intensity of Importance	*Definition*	*Explanation*
		practice
9	Extreme importance	The evidence favouring one activity over another is of the highest possible order of affirmation
2,4,6,8	Intermediate values	
1.1-1.9	If the activities are very close	May be difficult to assign the best value but when compared with other contrasting activities the size of the small numbers would not be too noticeable, yet they can still indicate the relative importance of the activities

Table 2.1. Scale of Relative Importance (Saaty, 1980)

When comparing elements A_i and A_j, A_i is said to be a_{ij} times more important than A_j, and the reciprocal value of the judgement is used to score the inverse comparison, that is, A_j is a_{ji} times more important than A_i. The self-comparison a_{ii} is always scored 1 (See formula 5).

$$a_{ij} = \frac{1}{a_{ji}}, \qquad a_{ii} = 1 \tag{5}$$

All judgements of the elements with respect to a given criterion are organised in a PC matrix (PCM). For *n* elements, the matrix is of size *n x n*, and because of the constraints in formula (5), only *n(n-1)/2* elements are provided by the DM.

$$A = \begin{bmatrix} 1 & a_{12} & a_{13} & \cdots & a_{1n} \\ 1/a_{12} & 1 & 1/a_{23} & \cdots & a_{2n} \\ 1/a_{13} & 1/a_{23} & 1 & \cdots & a_{3n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1/a_{1n} & 1/a_{2n} & 1/a_{3n} & \cdots & 1 \end{bmatrix} \tag{6}$$

There exists the possibility that the DM may provide inconsistent judgements.

For instance, if $a_{12}=3$, $a_{23}=2$, then we would expect that $a_{13}=6$. However, this is rarely the case. A PC matrix is said to be consistent or acceptable if its corresponding Consistency Ratio (CR[3]) is less than 0.1 (Saaty, 1980). There exist two types of consistency (Siraj, 2011), cardinal consistency (CC) and ordinal consistency (OC). For a matrix to be cardinally consistent, the condition $a_{ij}=a_{ik}a_{kj}$ must hold true for all i, j, k. The ordinal consistency refers to the transitivity of the preferences, if alternative A_i is preferred over A_j, and A_j is preferred over A_k, then A_i should be preferred over A_k. When this is condition is not met, the matrix is said to be ordinally inconsistent and it is not possible to find a weight vector that satisfies all preferences directions.

If the alternative A_i is preferred over A_j but the derived priorities are such that $w_j > w_i$, then a priority violation is present (Mikhailov, 1999). If the matrix is ordinally inconsistent, then priority violations may occur.

2.4 Elicitation Methods

To derive the weights or priorities from the PC matrix different methods may be used. For consistent matrices, generally all methods yield similar results only differing in intensities. Siraj (Siraj, 2011) analysed several methods and proposed two new methods based on graph theory and multi-objective optimisation: Enumerating All Spanning Trees (EAST) and Prioritisation using Indirect Judgements (PrInT). PrInT outperformed all other methods for inconsistent matrices. Based on their simplicity, however, the most common methods used to elicit priorities in AHP are Eigenvector (EV), Geometric Mean (GM) and Normalised Column Sum (NCS).

Some other methods are: Column-Row Orientation (Comrey, 1950), Direct Least Squares (DLS) (Chu, 1979), Weighted Least Squares (WLS) (Chu, 1979), Logarithmic Least Absolute Value (LLAV) (Cook, 1988), Logarithmic Least Squares (LLS) (Crawford, 1987), Fuzzy Preference Programming (FPP) (Mikhailov, 2000) and Two-Objective Prioritisation (TOP) (Mikhailov, 2006).

A study by Choo and Wedley (Choo, 2004) evaluated 18 different methods and

3 Also known as the Consistency Index (CI)

recommended GM and NCS for their easy calculation and good performance on consistent and inconsistent matrices.

2.5 Error Measures

A number of techniques exist to estimate the performance of a prioritisation method. The most common approach to calculating the total deviation (TD) or quadratic error between the priority vector and the provided judgements, is shown in formula (7).

$$TD = \sum_{i=1}^{n} \left(\sum_{j=1}^{n} \left(a_{ij} - \frac{w_i}{w_j} \right)^2 \right)^{\frac{1}{2}} \tag{7}$$

Another approach is to calculate the number of priority violations (NV). The formula (8) to calculate NV is as follows:

$$NV = \sum_{i=1}^{n} \sum_{j=1}^{n} I_{ij}, where$$

$$I_{ij} = \begin{cases} 1 & if\ (w_i > w_j)\ and\ (a_{ji} > 1) \\ 0.5 & if\ (w_i = w_j)\ and\ (a_{ij} \neq 1) \\ & or\ (w_i \neq w_j)\ and\ (a_{ij} = 1) \\ 0 & otherwise \end{cases} \tag{8}$$

2.6 An Illustrative Example

In order to illustrate the concepts of MCDM, AHP and PC consider the decision of purchasing of a new car (Bodin, 2004). Three alternatives have been identified, i.e. Avalon, Babylon and Carryon.

Consider that Price, Miles per Gallon (MPG) and Amenities are the three criteria that represent important attributes to the decision maker. Price and MPG can be considered quantitative criteria as they can be measured objectively. Amenities, in contrast, is a qualitative criterion, as it is a subjective attribute that

depends on the point of view of the DM. Furthermore, the DM identifies Prestige, Comfort and Style as Amenities' sub-criteria. The hierarchy tree representing this decision problem is shown in Figure 2.2.

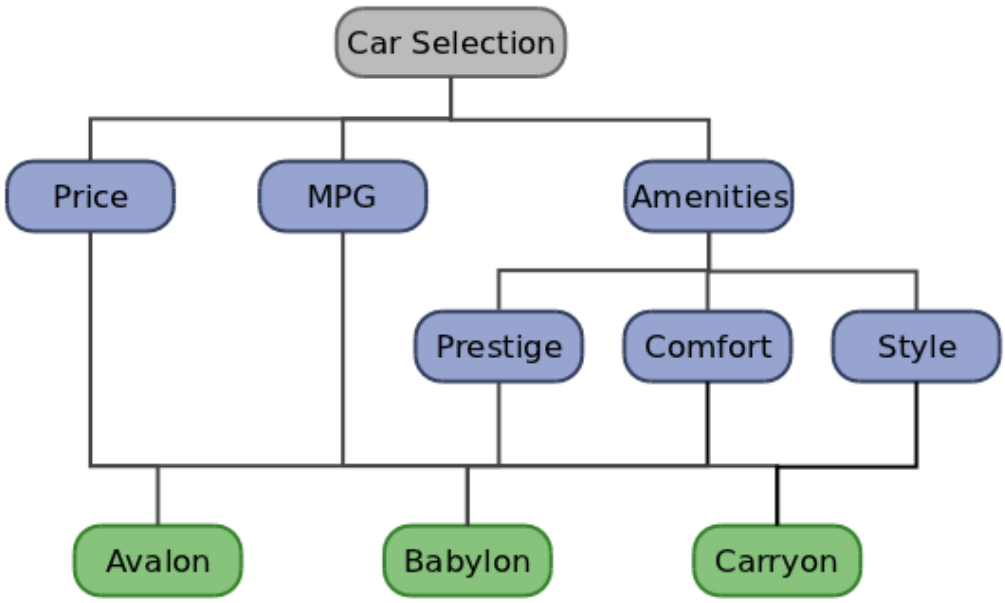


Figure 2.2. Car selection example

In order to make a decision, the DM has to give pairwise comparison judgements at every level of the hierarchy. In general terms, the attributes Price, Comfort and Prestige are of major importance to the DM, while he/she is less concerned about MPG and Style.

The DM gives the following judgements for the first level.

- Price has a strong or very strong importance when compared to MPG
- Price has a moderate importance when compared to Amenities
- Amenities has a moderate or strong importance when compared to MPG.

Table 2.1 is used in order to convert these verbal judgements into numeric values. As a consequence, Price is 6 times as important as MPG, Price is 3 times as important as Amenities and Amenities is 4 times as important as MPG, as represented in Table 2.2. The inverse comparison is evaluated with the

reciprocal value and the self-comparison is evaluated with 1.

	Price	*MPG*	*Amenities*
Price	1	6	3
MPG	1/6	1	1/4
Amenities	1/3	4	1

Table 2.2. Pairwise matrix for the goal

Using the Eigenvector method to elicit weights from the matrix the following values are obtained:

$$w_{price} = 0.644,\ w_{MPG}=0.085,\ w_{amenities}=0.271$$

Following the same approach, the DM evaluates the sub-criteria under Amenities and obtains the matrix shown in Table 2.3.

	Prestige	*Comfort*	*Style*
Prestige	1	1/3	3
Comfort	3	1	5
Style	1/3	1/5	1

Table 2.3. Pairwise matrix for Amenities

The corresponding weights are:

$$w_{prestige} = 0.258,\ w_{comfort}=0.637,\ w_{style}=0.105$$

Now that all criteria have been evaluated, the alternatives have to be compared under each criterion. Following the same procedure, the alternatives are assessed under criterion Price (Table 2.4), MPG (Table 2.5), Prestige (Table 2.6), Comfort (Table 2.7) and Style (Table 2.8). This time, the weights are included in each table.

	Avalon	*Babylon*	*Carryon*
Avalon	1	3	6
Babylon	1/3	1	4
Carryon	1/6	1/4	1
Weights	0.644	0.271	0.085

Table 2.4. Pairwise matrix for alternatives under criterion Price

	Avalon	*Babylon*	*Carryon*
Avalon	1	2	3
Babylon	1/2	1	2
Carryon	1/3	1/2	1
Weights	0.540	0.297	0.163

Table 2.5. Pairwise matrix for alternatives under criterion MPG

	Avalon	*Babylon*	*Carryon*
Avalon	1	1/6	1/3
Babylon	6	1	4
Carryon	3	1/4	1
Weights	0.091	0.691	0.218

Table 2.6. Pairwise matrix for alternatives under criterion Prestige

	Avalon	*Babylon*	*Carryon*
Avalon	1	1/5.5	1/8
Babylon	5.5	1	1/3
Carryon	8	3	1
Weights	0.064	0.279	0.657

Table 2.7. Pairwise matrix for alternatives under criterion Comfort

	Avalon	*Babylon*	*Carryon*
Avalon	1	1/7	1/4
Babylon	7	1	3.5
Carryon	4	1/3.5	1
Weights	0.077	0.679	0.244

Table 2.8. Pairwise matrix for alternatives under criterion Style

To calculate the global ranking of the alternatives formula (2) is applied to each node of the hierarchy to obtain the local contribution, and then the results are aggregated. For example, to obtain the local contribution of alternative Avalon to the node Amenities is as follows:

$$P_{avalon,amenities} = \delta_{avalon,prestige} * w_{prestige} + \delta_{avalon,comfort} * w_{comfort} + \delta_{avalon,style} * w_{style}$$

$$= 0.091 * 0.258 + 0.064 * 0.637 + 0.077 * 0.105$$

$$= 0.0723$$

And the global priority is calculated as:

$$P_{avalon} = \delta_{avalon,price} * w_{price} + \delta_{avalon,MPG} * w_{MPG} + \delta_{avalon,amenities} * w_{amenities}$$

$$= 0.644 * 0.644 + 0.540 * 0.085 + 0.0723 * 0.271$$

$$= 0.48$$

The priority for Babylon and Carryon are calculated in a similar way, resulting in the final ranking of alternatives as:

$$P_{avalon} = 0.48,\ P_{babylon} = 0.32,\ P_{carryon} = 0.20$$

The highest rated car is Avalon with 48%, followed by Babylon with 32% and Carryon with 20%. As a consequence, the outcome suggest that the DM should purchase the Avalon car.

Consider if the price of Babylon is reduced by a 20%, would Avalon be still the highest rated car? Similarly, how much would the price of Babylon have to be reduced, or the price of Avalon increased, to change the suggested recommendation? Or for example, if the DM is not completely sure about the

judgements between Style and Prestige, how would that affect the solution?

These kind of questions can be answered by conducting a sensitivity analysis (SA) on the results.

2.7 Sensitivity Analysis

The solution to a decision problem, the global ranking of alternatives, may not provide enough information to the DM to make a final decision. There are several reasons why a sensitivity analysis (SA) should be conducted on the results. For instance, the judgements for some criteria may be subjective or there may be uncertainty in the data that leads to the preference value. In addition, the preference judgements may come from a group decision where there are different opinions. Moreover, different prioritisation methods may yield different results for the same PC matrix; at the same time different performance scoring scales used in evaluating alternatives may produce different rankings (Steele, 2009). An SA provides more insight about the problem and in this way the DM should be able to make a more informed decision.

Methods to perform SA on AHP problems may be grouped into three main categories (Chen, 2008): numerical incremental analysis, probabilistic simulations and mathematical models.

2.7.1 Numerical Incremental Analysis

This approach involves changing the weight values and calculating the new solution. The method, also known as One-at-a-time (OAT), works by incrementally changing one parameter at a time, calculating the new solution and graphically presenting how the global ranking of alternatives changes. This is the most commonly used method in associated software tools ((SIMUL8, 2012), (MakeItRational, 2012), (ExpertChoice, 2012), (SAL, 2012), (IDS, 2010)) and, according to Chen and Kocaoglou (Chen, 2008), is also the most popular in the literature where AHP is used to solve problems.

As AHP uses WSM to aggregate local priorities, the global weights are a linear function depending on the local contributions. Given this property, the global priorities of alternatives can be expressed as a linear function of the local weights. Furthermore, if only one weight w_i is changed at a time, the priority P_i of alternative A_i can be expressed as a function of w_i using the following formula:

$$P_i = \frac{P_i'' - P_i'}{w_i'' - w_i'} (w_i - w_i') + P_i' \tag{9}$$

where P_i'' and P_i' are the priority values for w_i'' and w_i' respectively. With this method, only two iterations are necessary to produce a chart with the values of priorities of all alternatives for the range 0 to 1 of one of the weights, as shown in Figure 2.3.

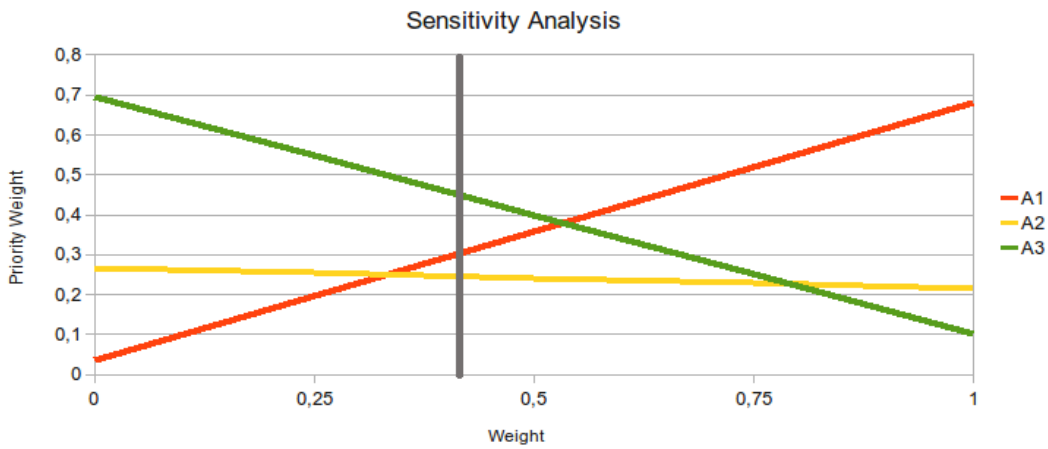


Figure 2.3. Numerical Incremental Sensitivity Analysis

The chart in Figure 2.3 is a common graphical tool to represent how the global ranking of alternatives (y-axis) is altered when the weight of one element (x-axis) is modified. The vertical grey line represents the current weight of the element in the solution. In this example, it can be noted that if the weight of the element is increased above 0.5, the ranking is reversed and the alternative A1 becomes the preferred choice.

Barker and Zabinsky (Barker, 2011) developed a MCDM model for a logistics

problem and proposed a method for finding the weights where a rank reversal[4] is produced in the model. The approach they used can be generalised to other problems. As only one weight w_k is changed at a time, all other weights w_l can be expressed as a function of w_k. Then, formula (2) can be expressed as a function of only one variable weight w_k. By solving the equation $P_i=P_j$, the value w_k where the ranking of both alternatives is the same (rank reversal) can be found.

Another approach proposed by Hurley (Hurley, 2001) multiplies the preference judgements in the PC matrix by a constant value and then the solution is recalculated using this new matrix. For constant values greater than 1, weights diverge; for values lower than 1, weights converge. Although this method will not produce a rank reversal, Hurley states that it may be useful to test the magnitude of the numerical weights elicited from the matrix.

2.7.2 Probabilistic Simulations

Simulation methods replace judgements in the PC matrix with values from probability distributions and perform a number of simulations to calculate the expected ranking of alternatives. As probabilistic input is used, the problem is no longer deterministic. In contrast with the previous method, this approach allows for changing more than one parameter at a time.

Butler et al. (Butler, 1997) proposed a method using Monte-Carlo simulations that allows random change of all weights simultaneously to explore the effect on the ranking. They presented three types of simulations:

1. Random weights

All criteria weights are generated completely at random in order to discover how the ranking of the alternatives changes under any conditions. To generate n weights, $n-1$ random numbers are generated in the interval 0-1 using a uniform random number generator. These numbers are sorted so that $1 > r_{n-1} > r_{n-2} > \ldots > r_2 > r_1 > 0$. In addition, let $r_n=1$ and $r_0=0$. The value for the weight w'_i is calculated as $w'_i = r_i - r_{i-1}$, for $i=1..n$. The vector $W'=[w'_1,\ldots,w'_n]$ will sum

4 Rank reversal occurs in line intersections in Figure 2.3

to 1 and is uniformly distributed.

By performing many repetitions of these steps a great number of times (5.000 iterations are used in the example), the entire domain of possible weight combinations can be explored.

For every iteration, the solution is calculated with random weights and the ranking of the alternatives is registered. After the simulation is finished, a statistical analysis can be performed to find information about the distribution of the ranking of alternatives. For instance, a box-plot chart may be created (see Figure 2.4) to depict information about the ranking of alternatives.

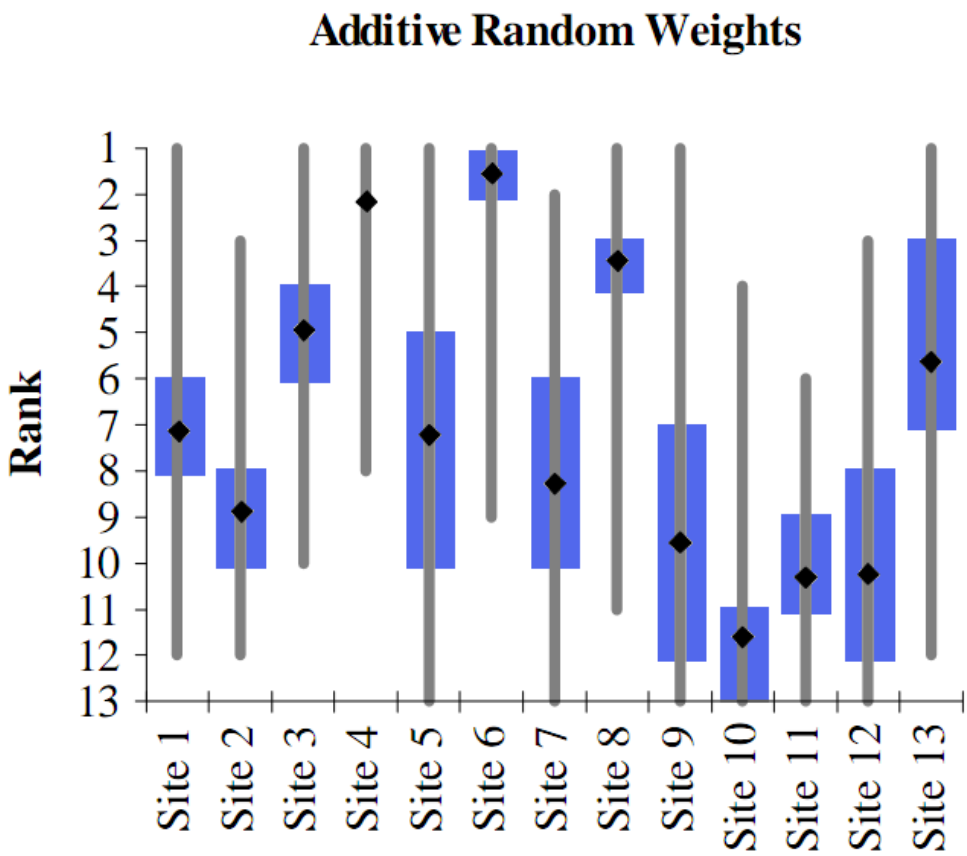


Figure 2.4. Box-plot Chart Presenting Simulation Results

The black diamonds correspond to the mean ranking, the blue box encloses quartiles Q1-Q3 (25%-75% of the samples had this ranking), and the minimum and maximum ranks are the endpoints of the grey lines (Butler, 1997).

2. Random weights preserving rank order

If the ranking of criteria weights are of importance and have to be preserved, then the procedure is similar to the previous one with the difference that the random weights are ranked according to the original criteria ranking and then are used to calculate the solution. The chart presented in Figure 2.4 is still valid when using this method.

3. Random weights from a response distribution

This type of simulation considers the current weights as means of probability distributions and the random weights for the simulation are generated from these distributions. Even though the random weights may be relatively close to the real weights, this approach may generate weights with a different rank order than the actual weights.

The formula to generate the random weights is:

$$w_i' = \frac{X_i}{X_i + \ldots + X_n} \quad , where\ X_i \sim Gamma(w_i, \beta) \tag{10}$$

The variation from the mean of the generated values can be controlled with the β parameter. Again, the procedure is similar, changing only the way random weights are generated.

A similar simulation approach, introduced by Hauser and Tadikamalla (Hauser, 1996), explores the effects of changing the judgements in the PC matrices. The method varies the preference values from the pairwise comparisons by creating an interval for each element of the matrix. The midpoint of the interval is the current value of the judgement and the width of the interval is defined by a constant *c* indicating the distance from the central point as a percentage. If a_{ij} is the preference value, then the interval I_{ij} may be represented as $I_{ij} = [a_{ij} - c\ a_{ij}, a_{ij} + c\ a_{ij}]$. The next step is to generate random numbers for each interval for each of the matrices in the model. Any probability distribution may be used. Hauser and Tadikamalla used the uniform distribution and the triangular distribution to test the method. Then, the random numbers have to be converted to the AHP scale using the formula (11):

$$a'_{ij} = \begin{cases} \frac{1}{2-r} & if\ r < 1 \\ r & if\ r \geqslant 1 \end{cases} \tag{11}$$

For every iteration, the solution is calculated with the random judgements and the ranking of the alternatives is registered. As with Butler's method (Butler, 1997), a statistical analysis can be performed to find information about the distribution of the ranking of alternatives.

In addition, Hauser and Tadikamalla proposed a formula to express the final ranking of the alternatives as statistical weights representing the expected weights for the given probability distribution, according to formula (12).

$$\begin{aligned} ES_i &= \sum_{k=1}^{n} p_{ik}(n+1-k), \quad for\ i = 1 \ldots \mathrm{n} \\ EW_i &= \frac{ES_i}{\sum_{k=1}^{n} ES_k}, \quad for\ i = 1 \ldots \mathrm{n} \end{aligned} \tag{12}$$

2.7.3 Mathematical Models

This group of SA methods uses mathematical models when it is possible to express the relationship between the input data and the problem solution. Mathematical models have better performance and are more efficient than the previous methods as they do not require iteration. Moreover, their results are much more accurate when using verified formulas.

Several authors have developed mathematical models for SA in AHP. Masuda (Masuda, 1990) studied how changes throughout the whole domain in the weights of criteria may affect the ranking of alternatives and proposed a sensitivity coefficient representing the possibility of rank reversal from these changes. Huang (Huang, 2002) found an inconsistency in Masuda's coefficient as a large value of the coefficient may not necessarily mean that a rank reversal will occur, and vice versa, a low value of the coefficient may produce a rank reversal. To overcome this, Huang proposed a new sensitivity coefficient and

demonstrated it reveals the sensitivity of an AHP model with more accuracy than Masuda's coefficient.

Erkut and Tarimcilar (Erkut, 1991) presented a method to analyse and visualise in the weight space (see Figure 2.5) the ranking of alternatives with respect to all possible combinations of weights for the first level of criteria. The method works by partitioning the weight space *T* into *n* subsets S_i, where *n* is the number of criteria. The weight space refers to all possible combinations of weights, as shown in formula (13):

$$(w_{1,} w_{2,} \ldots, w_n), \quad where\, 0 \leq w_i \leq 1 \;\wedge\; \sum_{i=1}^{n} w_i = 1 \tag{13}$$

For three criteria the weight space is a triangle-shaped plane with corners in *(1,0,0)*, *(0,1,0)* and *(0,0,1)*. For *n=3* criteria and *m* alternatives, Erkut and Tarimcilar provide a method for partitioning *T* into *m* subsets where in each subset some alternative has the highest ranking. The relative areas of these subsets indicate which alternatives are more likely to be selected.

However, there are limitations in their method. It does not provide a method for performing sensitivity analysis when the problem has more than one level of criteria. In addition, if more than three criteria are simultaneously considered, then visual representation is not possible and the weight space *T* is no longer a triangle, it becomes a convex polyhedron. Moreover, two extra algorithms have to be used to perform the partitioning (Balinski, 1961) and the calculation of the volumes (Cohen, 1979) (instead of areas) of the subsets.

Trantaphyllou and Sánchez (Triantaphyllou, 1997) investigated the sensitivity analysis for problems with one level of criteria by examining the impact of changes in the weights of criteria and changes in the weights of alternatives with respect to a single criterion at a time. They proposed a method to identify the most critical criterion (for criteria weights and for alternative weights) and the quantity by which it needs to be modified so that the top-ranked alternative will change. In addition, they introduced a sensitivity coefficient as an indicator of how sensitive to changes are the elements in the decision problem.

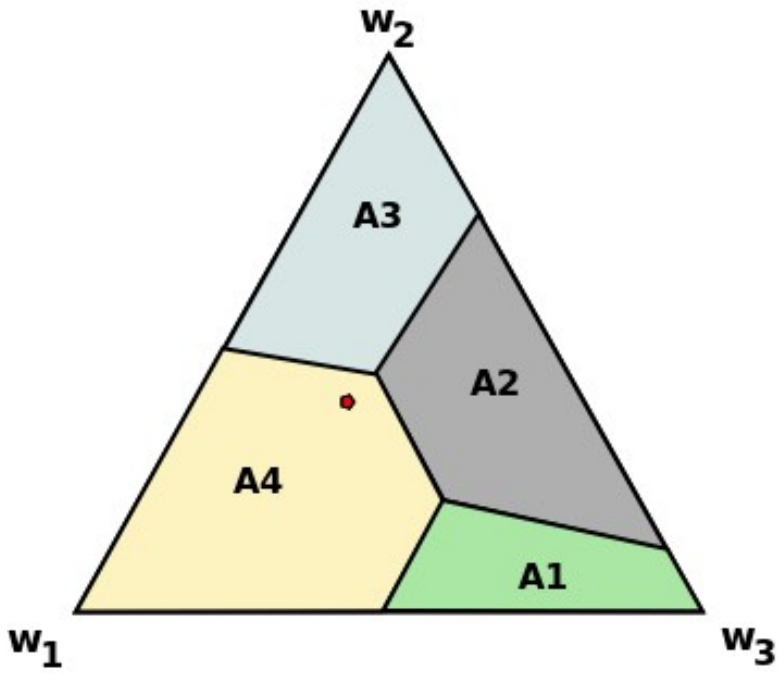


Figure 2.5. Ranking of Alternatives in the Weight Space

The triangle represents all possible combination for weights w_1, w_2, w_3. The red dot represents the actual weights in the solution. Each area A_i is the top-ranked alternative for the combination of w_1, w_2 and w_3 (Erkut, 1991).

Another study to identify the most sensitive criterion was presented by Armacost and Hosseini (Armacost, 1994). They proposed a method to find which criteria are determinant in the final ranking of alternatives using Direct Dual Questioning Determinant Attribute analysis (DQDA). The DQDA approach identifies the determinant criteria by measuring the *importance* ratings for the criteria and the *difference* ratings among the alternatives with respect to each criterion. The product of these two ratings for each criterion is the *determinance* score. The criteria with the highest *determinance* scores are said to be the determinant attributes of the problem.

Taji and Matsumoto (Taji, 2006) worked on the PC matrix sensitivity. They proposed a method to estimate intervals in which the PC values of a matrix can move without exceeding a consistency index (CI) threshold without causing recalculation of the eigenvalue and eigenvector of the matrix. As the method does not need to recalculate the eigenvalue and eigenvector of the matrix, a quick estimation of the interval is possible.

Chen and Kocaoglu (Chen, 2008) proposed an algorithm to examine the impact of single or multiple changes to the weights of criteria or alternatives at any level of the AHP hierarchy. Their approach finds the allowable range of changes in order to prevent a rank reversal. In addition, two sensitivity coefficients are presented and the most critical criterion in each level of the hierarchy is identified.

Iryanto (Iryanto, 2006) investigated the sensitivity of Interval AHP (IAHP). In IAHP, judgements in the PC matrix are provided as intervals instead of single values. Weights are elicited from the PC matrix using linear programming by minimising the error between the weight vector and the geometric mean of each interval. Iryanto concluded that when changes are introduced to an interval, as long as the limits of the new interval remain inside the original interval, the ranking is preserved.

Chen and Li (Chen, 2011) have recently developed a method to conduct SA using the column-row orientation elicitation method (Comrey, 1950). This algorithm calculates the allowable range of judgement perturbations induced in a PC matrix at any level of the decision hierarchy in order to keep the current ranking of alternatives unchanged.

2.8 Summary

This chapter has presented an overview of MCDM, AHP and PC. In addition, a discussion of SA was presented and its different types were covered.

Sensitivity analysis methods may be grouped into three main categories: numerical incremental analysis, probabilistic simulations and mathematical models. The first method is the most popular because of its simplicity and easy implementation. However, the other two groups of methods may provide more insights on a decision problem as they allow simultaneous analysis on more than one decision element.

Chapter 3
Requirements for AHP Software

This chapter focuses on common criteria for evaluating AHP tools. The focus is on features for the AHP and PC techniques described in the previous chapter. The aim is to create a framework to analyse available software tools and examine to what extent they comply with these features.

3.1 Requirements for Evaluating Software

There is limited literature on this subject. A number of studies ((Buede, 1992), (Buede, 1996), (Weistroffer, 1997)) have reviewed MCDM software in terms of general attributes such as platform, price and methodology[5]. Other study (Ossadnik, 1999) evaluated AHP software based on the model ISO/IEC 9126 (Caia, 2011). However, no study appears to have focused on evaluating AHP features and SA.

From the material presented in Chapter 2, the main features expected in an AHP software tool are described below (Sections 3.1.1 and 3.1.2). They are divided into two groups: support for AHP and PC, and support for SA.

3.1.1 Support for AHP and PC

AHP software should support hierarchical models, pairwise comparison, numeric and verbal scales, consistency analysis, methods for elicitation and provide performance measures.

A1. Hierarchical models: a tool should allow a hierarchy model with arbitrary levels of criteria and number of alternatives. In this way, any decision problem can be modelled as a hierarchy and solved using AHP.

A2. PC for criteria and alternatives: pairwise comparison should be available to provide judgements for criteria and for alternatives with respect to

5 Methodology refers to the alternative ranking method used, e.g. AHP (Saaty, 1980), MAUT (Wallenius, 2008), ELECTRE (Roy, 1968).

a criterion. This means that at every level of the hierarchy the weights of the elements should be elicited from pairwise comparisons.

A3. Numerical and verbal scales: the scale to provide the judgements should be both numerical and verbal, as shown in Table 2.1.

A4. Consistency analyser: to assist the DM to improve consistency of the PC matrix, its consistency index (CI), ordinal consistency and cardinal consistency should be displayed.

A5. Prioritisation methods: a software tool should support more than one prioritisation method so DMs can evaluate different methods and choose a solution according to their needs.

A6. Error measures: Error measures (TD, NV) must be provided so the DM can get an indicator of the deviation of the solution from the provided judgements.

3.1.2 Support for Sensitivity Analysis

As discussed in Section 2.7, there are several approaches to conduct an SA in AHP. We conclude that an AHP tool should provide each of the three forms of SA identified: numerical incremental analysis, probabilistic simulation and mathematical models. In addition, the most sensitive criteria should be presented.

B1. Numerical incremental analysis: a module for performing the one-at-a-time sensitivity analysis should be provided, along with a graph similar to the one presented in Figure 2.3 to display the sensitivity of a criterion.

B2. Probabilistic simulation: a module for performing probabilistic simulations should be provided in order to find the sensitivity of alternatives to simultaneous changes in all weights, including a graph similar to the one depicted in Figure 2.4.

B3. Mathematical models: using the mathematical approach to SA the threshold values for the weights that produce a rank reversal should be presented in a graph similar to the one shown in Figure 2.5.

B4. Most sensitive criteria: along with the results of the SA, the most sensitive criteria should be presented and sensitivity coefficients should be included so the DM can have an estimation of how sensitive is each criterion.

B5. Number of simultaneous changes: the number of elements that can be changed at the same time to analyse the results; more than one simultaneous change should be supported.

3.2 Software Products

In this section, four commercial or research prototype tools that support AHP and PC are analysed in terms of the framework described above: Expert Choice (ExpertChoice, 2012), MakeItRational (MakeItRational, 2012), PriEsT (Siraj, 2012a) and Web-HIPRE (SAL, 2012).

For each feature (A1-A5, B1-B5) a score between 0 and 3 will be assigned: 0 if the feature is not present, 1 if it is poorly implemented, 2 if it is moderately supported, and 3 if it is fully supported. The overall score for a tool will be the sum of the individual scores.

3.2.1 Expert Choice

Expert Choice ((ExpertChoice, 2012), (Ishizaka, 2009), (Chan, 2007)) is a commercial desktop-based application for Windows that enables DMs to prioritise and evaluate criteria and alternatives using AHP. The analysis is detailed in Table 3.1. It is worth noting that Expert Choice only supports the basic form of SA, i.e. changing one parameter at a time and presenting the results. Its overall score is 13 out of 33.

Feature	*Score*	*Description*
A1	3	Expert Choice provides a GUI to build an arbitrary-depth hierarchy (student version is limited to 3 levels). Elements can be added, edited and removed from the model
A2	3	The software supports pairwise comparison in all levels of

Feature	*Score*	*Description*
		the hierarchy
A3	3	The judgements may be entered using a numeric or verbal scale
A4	1	Only the consistency index is displayed. No information about cardinal or ordinal inconsistency is presented
A5	0	Only the eigenvector method is included in the software. There is no option to change the method or add new ones
A6	0	No error measures are presented when calculating the weight vector
B1	2	Numerical incremental analysis is supported with three graphs that allows to change the weights of one criterion. It is not possible to change the weights of alternatives with respect to one criterion, only criteria weights
B2	0	Expert Choice does not support probabilistic simulations
B3	0	There is no support for identifying the threshold values for rank reversal. It has to be done manually using the numerical incremental analysis
B4	0	No sensitivity coefficient is provided and the most sensitive elements are not identified
B5	1	Only one parameter can be modified at a time

Table 3.1. Expert Choice Software Evaluation

3.2.2 MakeItRational

MakeItRational (MakeItRational, 2012) is a commercial web-based application for decision making with a user-friendly interface. It requires the Silverlight plugin from Microsoft to run on a supported web-browser. The analysis is detailed in Table 3.2. Its overall score is 12 out of 33.

Feature	*Score*	*Description*
A1	3	MakeItRational provides a GUI to build an arbitrary-depth hierarchy. Elements can be added, edited and removed from the model
A2	3	Pairwise comparisons – or direct rating- are supported in all levels of the hierarchy
A3	2	Judgements are entered using a numerical scale, however, a description is available for each number of the scale
A4	3	The software includes the consistency index and hints for helping the DM to improve the ordinal and cardinal consistency when the judgements are inconsistent
A5	0	Only the eigenvector method is included in the software. There is no option to change the method or add new ones
A6	0	No error measures are presented when calculating the weight vector
B1	1	Numerical incremental analysis is supported with one static graph similar to the one in Figure 2.3. It does not allow to change the weights of criteria or alternatives
B2	0	No support for probabilistic simulations
B3	0	There is no support for identifying the threshold values for rank reversal. It has to be done manually using the graph from the numerical incremental analysis
B4	0	No sensitivity coefficients are provided and the most sensitive elements are not identified
B5	0	No changes are possible. The sensitivity graph is static

Table 3.2. MakeItRational Software Evaluation

3.2.3 PriEsT

PriEsT is an open source tool developed at the University of Manchester (Siraj,

2012a) that provides decision support based on AHP and PC. PriEsT is implemented in Java and is currently available for Linux and Windows. The overall score for this tool is 16 out of 33. The results from the analysis are presented in Table 3.3.

Feature	*Score*	*Description*
A1	2	PriEsT supports an unlimited number of decision elements and levels in the hierarchy. Elements can be added and edited, but not removed from the model
A2	3	Pairwise comparisons are supported in all levels of the hierarchy
A3	2	Judgements can be entered using only a numerical scale - a verbal scale is not supported
A4	3	The consistency index, cardinal and ordinal consistency measures are displayed as well as hints for helping the DM to improve overall consistency of the judgements
A5	3	PriEsT supports 11 prioritisation methods which can be selected to derive weights
A6	3	Error measures TD, NV and TD2 (total deviation using indirect judgements (Siraj, 2011)) are presented
B1-B5	0	PriEsT has no support for sensitivity analysis

Table 3.3. PriEsT Software Evaluation

3.2.4 Web-HIPRE

Web-HIPRE (SAL, 2012) is a free web-based application for MCDM implemented as a Java applet. It was the first decision tool available online (Mustajoki, 2000). Table 3.4 presents the analysis of the software according to the previously defined features. The overall score is 14 out of 33.

Feature	*Score*	*Description*
A1	3	Hierarchies of up to 17 levels can be defined using the web interface – this is enough for most problems. Elements can be added, edited and removed from the model
A2	3	Pairwise comparisons are supported in all levels of the hierarchy
A3	3	Judgements can be entered using a numerical or verbal scale
A4	1	Only the consistency index is displayed. No information about cardinal or ordinal inconsistency is presented. No hints for improving the consistency of the judgements
A5	0	Only the eigenvector method is included in the software when working with AHP. There is no option to change the method or add new ones
A6	0	No error measures are presented when calculating the weight vector
B1	3	Numerical incremental analysis is supported with a dynamic graph similar to the one in Figure 2.3. It allows to change the weights of one element (criterion or alternative)
B2	0	No support for probabilistic simulations
B3	0	There is no support to identify threshold values for rank reversal. This has to be done manually for each criterion using the graph from the numerical incremental analysis
B4	0	No sensitivity coefficients are provided and the most sensitive elements are not identified
B5	1	Only one parameter can be modified at a time

Table 3.4. Web-HIPRE Software Evaluation

Out of a maximum possible score of 33, PriEsT Is ranked first with a score of 16 followed by Web-HIPRE with 14. The analysis shows that all tools have similar support for AHP and PC, the most significant difference is that only PriEsT provides more than one elicitation method and presents error measures with the results. PriEsT is the only tool with no support for SA; other tools provide a basic form of SA using the numerical incremental analysis. No tool provides sensitivity coefficients or identification of the most sensitive criteria. The lack of these features contributed to the relative low score of all the software tools.

As PriEsT has obtained the highest overall score, it has been selected to be enhanced by including a module for conducting the three types of SA - features B1-B5 - and features A1 and A3, as it scored 2 out of 3 for these two criteria. A brief introduction to PriEsT is given next.

3.3 An Overview of PriEsT

PriEsT (Priority Estimation Tool) is a decision support tool for AHP and PC that helps DMs to identify inconsistency and revise their judgements by providing several visual aids and consistency measures (Siraj, 2011).

The visual aids include displaying the most inconsistent set of three judgements, identification of three-way cycles, and displaying congruence and dissonance values. Congruence measures the cardinal inconsistency of the judgements while dissonance the ordinal inconsistency.

In addition, PriEsT provides three error indicators with the weights: TD, NV and TD2 (Total Deviation using Indirect Judgements), in this way decision makers are more informed about the solutions presented to them.

While other tools usually implement either Eigenvector or Normalised Column Sum, PriEsT supports 11 methods for eliciting weights, including two new methods, EAST and PrInT, which are briefly discussed below.

- EAST (Enumerating All Spanning Trees). The EAST (Siraj, 2012b) elicitation method is based on graph theory. It represents all possible preferences of the DM and allows elicitation of weights from incomplete

PCMs.

- PrInT (Prioritisation using Indirect Judgements). The PrInT method (Siraj, 2011) uses an optimisation approach to elicit weights by minimising TD, NV and TD2 simultaneously. PrInT produces a wide range of non-dominated solutions giving DMs the flexibility to select any of these solutions according to their requirements. Moreover, for inconsistent PCMs, PrInT outperforms other elicitation methods (Siraj, 2011), providing solutions with minimum priority violations whilst the generated weights are as close as possible to both direct and indirect judgements.

3.4 Summary

Based on the background literature review, this chapter has presented a list of requirements covering both AHP and SA that were then used to evaluate four decision support tools. Despite PriEsT not supporting SA, it obtained the highest overall score and, therefore, is selected as the tool to be enhanced by including a module for SA.

The next chapter describes the re-engineering process carried out in PriEsT to be able to include SA and other features, and Chapter 5 covers the implementation details and integration of the SA algorithms in PriEsT.

Chapter 4
Re-engineering PriEsT

This chapter covers the re-engineering process that was performed in order to improve the tool and facilitate the implementation of the sensitivity analysis (SA) algorithms as described in the next chapter. This process involved the study of the features, design and structure of the source code of PriEsT, the preparation for re-engineering, the details of the new design and the implementation of the new version of PriEsT.

4.1 Motivation and Context

In software engineering, it is important to make a clear separation between the domain model (model) and the user interface (UI) objects (view) - normally known as the Model View Controller (MVC) pattern (Larman, 2004). The motivation for this separation includes support for cohesive model definitions that focus on the domain processes rather than on UI objects, minimise the impact of changes in model requirements or UI, allow creation of new views connected to an existing domain model, allow the model to be ported to a different UI and allow the execution of the model without a UI.

Enhancement of a software too is a notoriously difficult exercise (and PriEsT has had hundreds of downloads in the last year) as it requires maintenance of as much of a highly cohesive and loosely coupled structure as possible to allow for further possible enhancement. Further, the original version of PriEsT was a prototype linking a number of individual research contributions rather than developed as a usable MCDM AHP tool. Hence, before implementing the SA module, it is sensible and good practice to analyse the PriEsT system from a re-engineering perspective and where necessary bring it to a state where further development becomes easier and maintenance of the source code is more effective and efficient. To achieve this objective, the internal software architecture of PriEsT has been analysed, redesigned as appropriate, with part

of the source code being re-written whilst preserving its functionality and the look and feel of the UI.

4.2 Current State of PriEsT

PriEsT is an open source tool developed at the University of Manchester (Siraj, 2011) that provides decision support based on AHP and PC. PriEsT is implemented in Java and is currently available for Linux and Windows. More details about PriEsT can be found in Section 3.3.

4.2.1 Limitations

The software is implemented using the Qt Jambi library (qt-jambi, 2011). This library uses native methods to communicate with the native QT library. For this reason, even though PriEsT is written in Java, it is not completely platform independent as it needs the native QT libraries and the Java platform-specific QT bindings to run.

Although PriEsT was designed based on the MVC architecture as shown in Figure 4.1, a detailed analysis of the design and source code of PriEsT has shown that there is only a partial separation of functions between the GUI and the logic for AHP problem-solving. The core classes (the PriEsT Engine) provide the basis functions such as priority vectors, elicitation methods, etc., but it is the GUI interface that contains the logic to execute the elicitation methods on the different criteria in a hierarchy tree. Moreover, the GUI stores all the elicitation results in an in-memory database and has the algorithms to calculate the aggregated rankings and errors (TD, NV, TD2) from these results.

In addition, because of dependencies with the QT libraries the code is hard to reuse. For instance, if another front-end would be implemented, only part of the PriEsT Engine could be used, but the algorithms for aggregating results and calculating the inconsistencies would have to be rewritten.

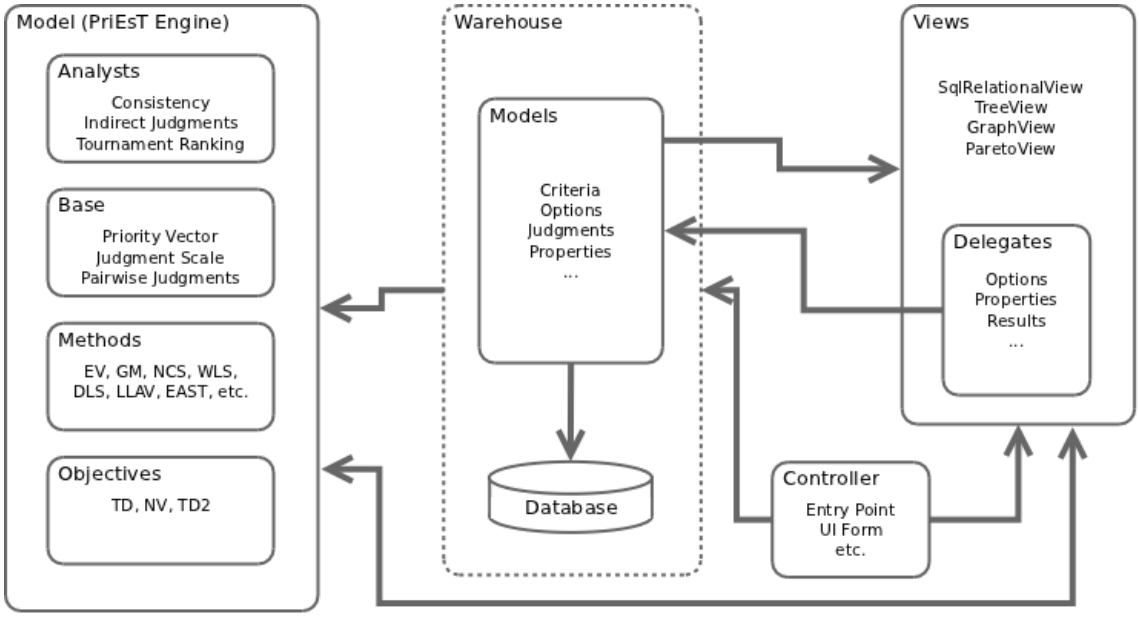


Figure 4.1. Old MVC Architecture of PriEsT

Adapted from (Siraj, 2011)

Other minor issues are:

- The names of classes and methods do not follow the Java name conventions making the code harder to read and understand.
- As PriEsT needs the QT libraries to run, the distributable package necessary to execute PriEsT is over 20Mb. Although this is not really a problem, it may be an inconvenience that could prevent people from downloading the tool.

These non-functional limitations are inconvenient and make further development more difficult and time consuming.

4.3 New Requirements

In order to overcome the previous limitations and include the features discussed in Chapter 3, new requirements for PriEsT are introduced. These requirements are grouped into two categories: functional requirements (FR)

and non-functional requirements (NR).

4.3.1 Functional Requirements

- FR-001: PriEsT already supports adding and modifying decision models, but there is no option to remove a criterion or alternative forcing the user to create a new model. The new version of PriEsT will allow the user to remove any decision element from the model.
- FR-002: Solving a decision problem in PriEsT is a two step operation, the user first have to elicit the nodes and then calculate the ranking of alternatives using the results from one elicitation method. The new version of PriEsT will calculate the ranking of alternatives automatically when the user elicit the nodes and will present the ranking for all the active elicitation methods.
- FR-003: A module for conducting one-at-a-time SA will be included. The user will select a criterion and a child element. By changing the weight of the child element the new ranking of alternatives will be presented. In addition, the points where rank reversal occurs will be identified.
- FR-004: A module for conducting probabilistic simulation SA will be included. The criteria and the probability distribution for the simulation will be user selectable. This module will present the ranking of alternatives that is likely to occur under the conditions selected by the user.
- FR-005: A module for conducting a SA based on the mathematical model approach will be included in PriEsT. Using this feature, the user will be able to visualise the threshold values of weights that produce a rank reversal. The module will allow the number of weights at any level in the hierarchy tree to be changed.
- FR-006: PriEsT will include a sensitivity coefficient along with the results of the SA so the user can have an estimation of how sensitive each criterion is and help to identify the most sensitive criterion.

4.3.2 Non-Functional Requirements

- NR-001: The architecture of the new version of PriEsT must have a clear Model-View separation. All the AHP related logic must be clearly separated from the front-end to facilitate further development and the implementation of different front-ends.
- NR-002: The algorithms and logic for working with AHP problems must be encapsulated in a library as a JAR file (Java Archive file) and the front-end must be encapsulated in a different JAR file. In this way, the process for improving, fixing bugs and adding features to the PriEsT library can be done independently from the front-ends using it. In addition, different front-ends can be implemented that make use of the PriEsT library with no need to modify the source code of the library.
- NR-003: The implementation of the PriEsT library and front-end must be done using pure Java with no dependencies on native libraries. All QT dependencies must be removed. This will allow PriEsT to be multi-platform, able to run in any operating system provided it has the Java Virtual Machine (JVM).
- NR-004: A web version similar in features to the desktop version must be implemented using the PriEsT library as the core. The web version must be implemented as a Java Applet able to run in a web browser that supports Java.
- NR-005: A mobile version of PriEsT must be implemented that can run on Java enabled smart phones. This version must use the PriEsT library as the core.

4.4 Preparation for Re-engineering

Before implementing the functional requirements in PriEsT a re-engineering process is required to improve its modular structure, ensure a model-view separation and generally provide better coherence and lower cohesion. Major

steps in the re-engineering process are source code translation, program structure improvement and program modularisation (Sommerville, 2000). As part of the source code translation, the QT Library will be replaced by the Java Swing framework to remove the dependency on native libraries and in this way improve portability. The structure improvement will involve a clear model-view separation to improve cohesion and code readability. Finally, to improve program modularity, all AHP related code will be grouped together in a Java library making it easier to re-use and maintain, hence, reducing time and effort to implement new features.

4.4.1 Source code re-use

As the source code will be re-structured, some classes will be discarded, others modified and some will be re-used with minor modifications.

PriEsT is currently made up of 66 classes, of which 41 form the PriEsT Engine and 25 form the GUI. After some modifications, 22 classes from the 41 classes of the PriEsT engine can be re-used to construct the PriEsT library. From the 25 classes of the GUI, only 2 can be re-used after some modifications because the QT library is incompatible with the Swing framework.

4.5 New Design

To maintain a clear separation of concerns in reference to requirements NR-001 and NR-002, the design and implementation has been separated in two libraries: PriEsT library and desktop application.

4.5.1 PriEsT Library

The PriEsT library contains the core of the software. It has been designed to be completely independent of the front-end, re-usable and easy to use.

The library consists of several building blocks, as shown in Figure 5.1. Additional details and a complete UML class diagram can be found in Appendix A and some code-snippets showing how to use the library Appendix C. Each building block

encapsulates related functionality and interact with others to provide the core features of PriEsT. These building blocks are described below.

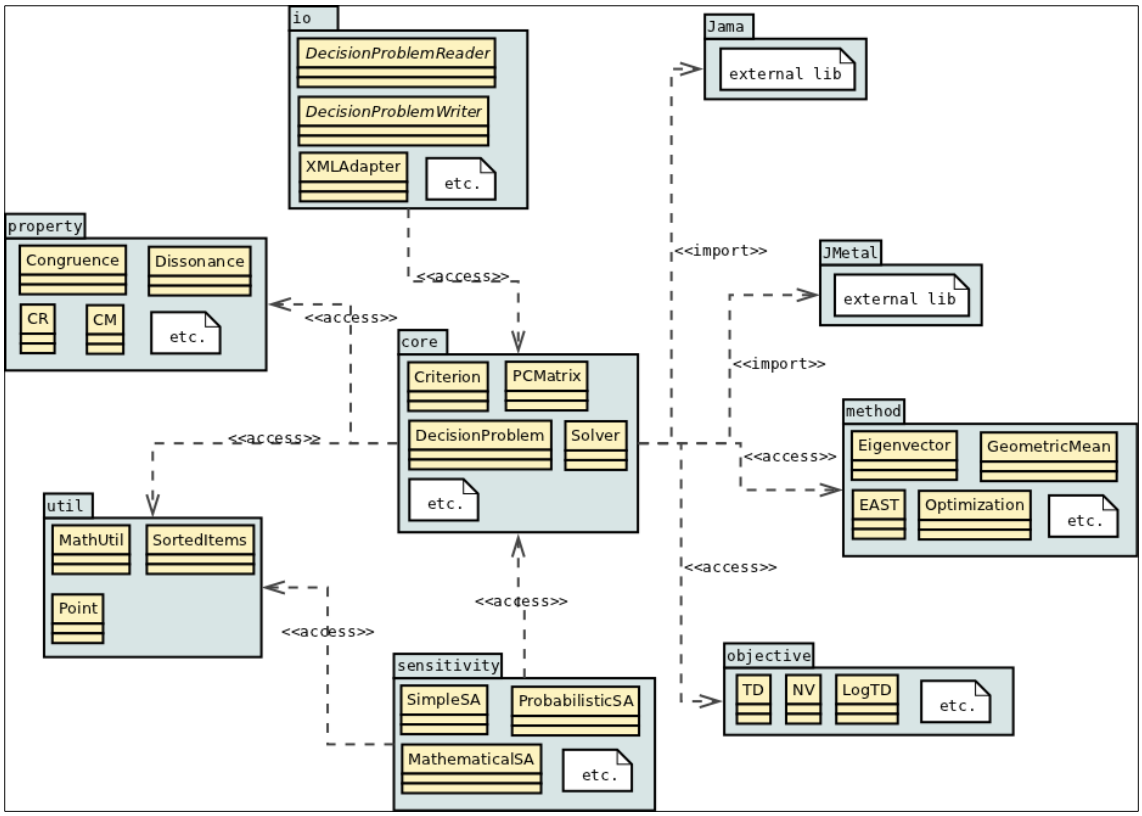


Figure 4.2. PriEsT Library Design

4.5.1.1 Core

This building block contains all the foundation classes to support AHP and PC. The classes in this package provide functionality for defining a decision problem, criteria, PCMs and solving the problem to obtain the ranking of alternatives. This block is the public interface of the library and provides an easy-to-use set of classes for accessing the library.

4.5.1.2 IO

This package contains the classes necessary to import and export decision problems from XML files and other file formats. Two Interfaces and an abstract class allow the implementation of more classes to support additional formats without the need to modify existing code.

4.5.1.3 Method

This package contains each of the elicitation methods (EV, GM, etc.) mentioned in Section 2.4. A base class provides the common functionality for each method. All non-optimisation methods (EV, GM, NCS and EAST) are implemented using a separate class. The optimisation-based methods[6] (DLS, WLS, LLAV, LLS, FPP, TOP and PrInT) are implemented using a single class that takes one or more objectives according to their optimisation goal.

4.5.1.4 Objective

This building block defines all the objectives used in the optimisation methods, such as TD (total deviation), NV (number of violations), TD2 (total indirect deviation), LogTD (logarithmic deviation), etc.

4.5.1.5 Property

This block implements all the properties that can be calculated from a pairwise comparison matrix. Five properties are implemented, i.e., dissonance, congruence, cycles, CR and CM. Creating a package to encapsulate the properties outside of the PCM makes it easier to modify or add new properties without affecting the existing code.

4.5.1.6 Sensitivity

This package contains all the classes required to implement the SA algorithms. More details about this implementation are given in Chapter 5.

4.5.1.7 Util

This building block contains classes with general operations shared by all building blocks, such as mathematical formulas or sorting algorithms.

4.5.2 Desktop Application

In relation to requirement NR-003, the UI is designed using the default user-

6 More information about these methods can be found in (Siraj, 2011)

interface framework in Java, Swing. In this way, all dependencies on external libraries are removed allowing the application to run on any operating system where the JVM is available.

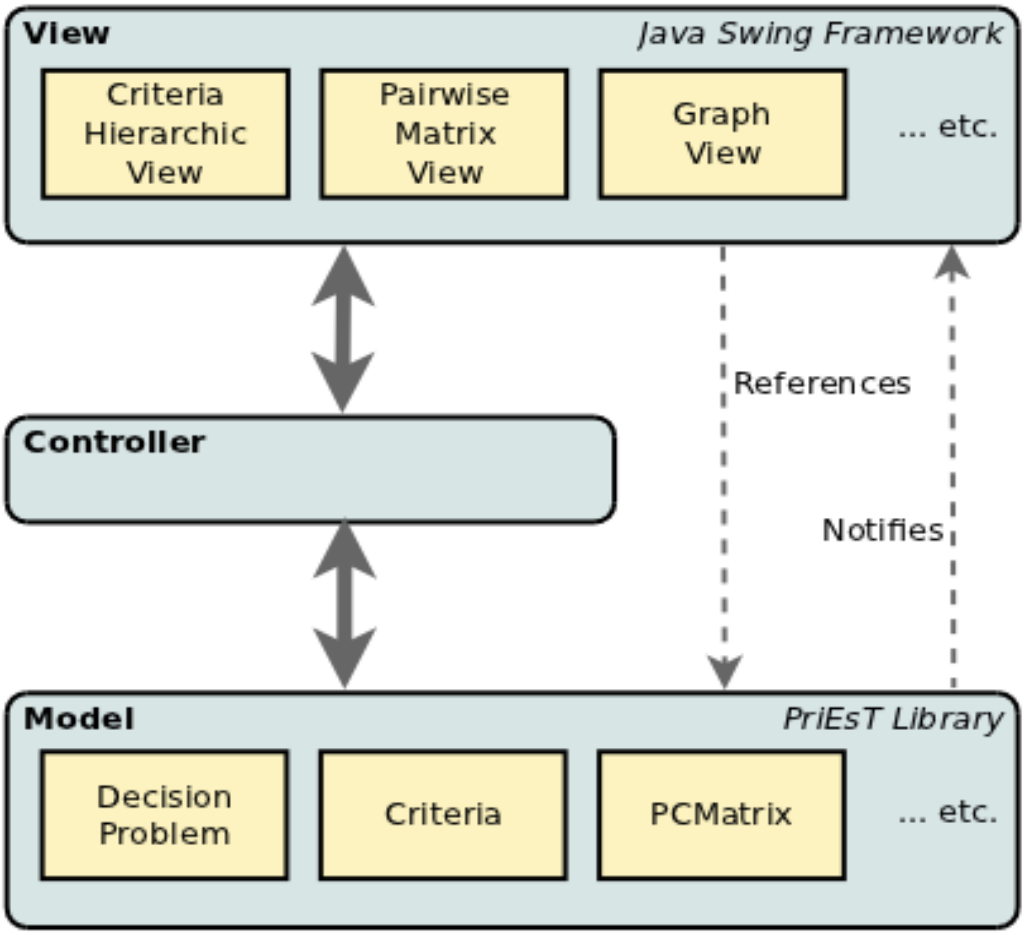


Figure 4.3. MVC Architecture

The design of the application is based on the MVC architecture as shown in Figure 4.3. The view (visible elements of the application, e.g., hierarchic model, pairwise comparison matrix, charts, etc.) handles the presentation and interaction with the user. When the user requests an operation on the view, it is sent over to the controller in order for it to handle it. In addition, when a view needs to be updated (for instance, an algorithm has finished execution), the model -by using the observer pattern- may notify the controller or the view with the new status.

The controller is the layer responsible for coordinating all activities between the view and the model. The controller acts as an entry point to the model in a way that decouples the UI from the implementation details of the model.

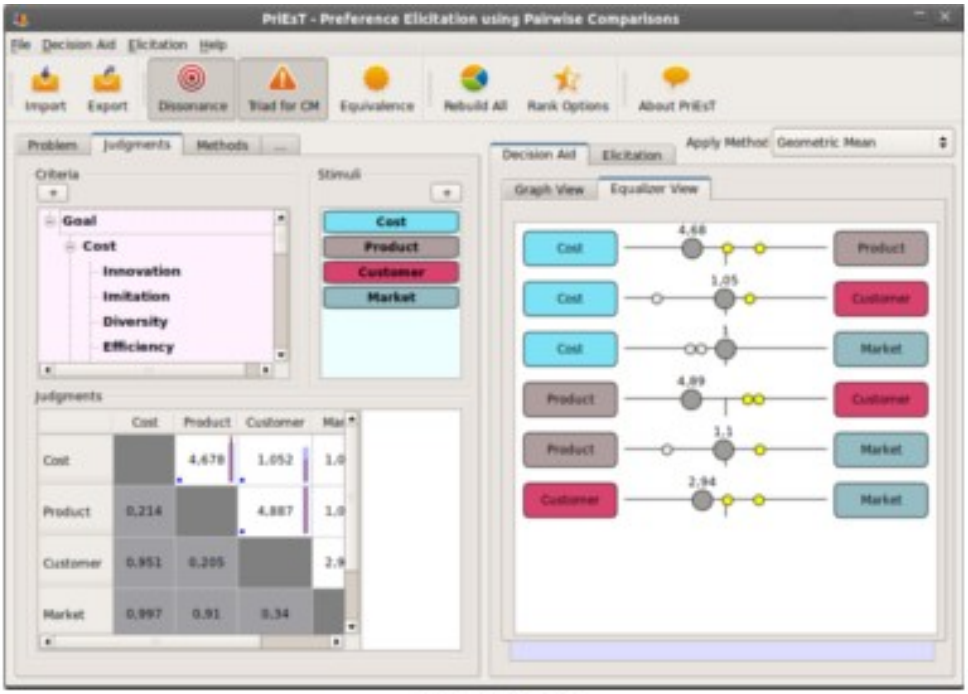
The model corresponds to the PriEsT library. It stores the state of the decision problem, criteria, pairwise comparisons, results, etc. The model is independent of the controller and view which results in a design with low coupling.

Figure 4.4 shows two screen-shots of PriEsT. Figure 4.4a is a snapshot of the previous version of PriEsT implemented with QT. Figure 4.4b shows the new re-engineered version using Java Swing. As can be seen from the screen-shots, both versions look very similar, although the architecture has changed significantly. In the new version the button “Rank Options” was removed as part of the implementation of requirement FR-002, and a new tab was added to the right panel for sensitivity analysis.

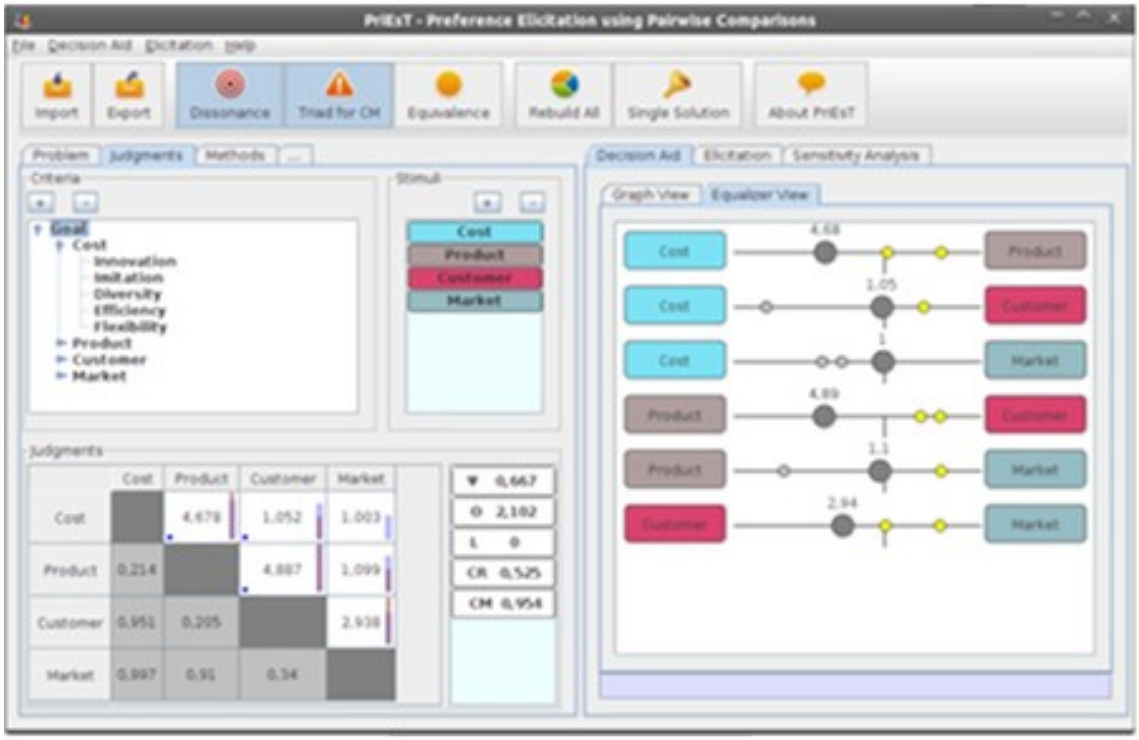
In addition, the size of the distributable package have been decreased significantly. PriEsT and all required libraries are now 1Mb in size, the previous version was over 20Mb.

4.6 Implementation Methodology

To accomplish the main goal of this project - to enhance the PriEsT tool by including a module to conduct sensitivity analysis - the requirements, design and re-engineering plan have been laid out. Now, in order to implement all requirements, the evolutionary prototyping development methodology (Larman, 2004) was selected. This method is capable of handling requirements changes by developing an initial prototype which implements well-understood requirements. Then, this prototype is validated with the stakeholders and new requirements discussed. Each successive prototype implements more functionality, and at the same time, refines previous work following stakeholder feedback.



(a) QT version



(b) New version

Figure 4.4. PriEsT UI

In addition, good practices from agile methodologies were used, such as Test Driven Development (TDD). Complex requirements, such as formulas and

algorithms to perform SA, were implemented using TDD. That is, a set of test units were created before implementing the algorithms and then were used to build the actual methods. These test units helped to ensure that mathematical formulas and algorithms are correctly implemented. Some of these tests are briefly described in Chapter 7 and Appendix B.

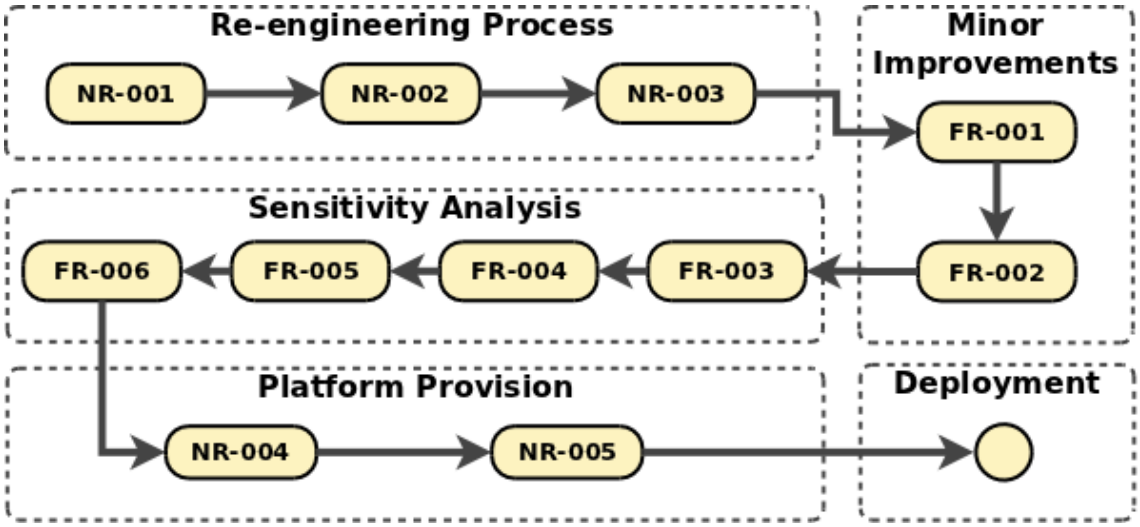


Figure 4.5. Implementation Steps

The order in which the requirements were addressed is as shown in Figure 4.5. First, the re-engineering process was performed. A new design with a clear model-view separation was created (NR-001) and the PriEsT library was implemented (NR-002) and tested without a front-end. Then, a front-end with the same look and feel as the previous version but using only Java libraries was implemented (NR-003). After this new version was successfully tested the work on the new features was carried out. First, requirements FR-001 and FR-002 were completed. The SA related requirements were addressed next, and finally web and a mobile version of the application were implemented. The last step was deployment on a mobile phone and publishing the applet version in a web page. Details of the implementation of SA can be found in Chapter 5 and web and mobile versions in Chapter 6.

4.7 Implementation Platform, Languages and Tools

All the different components of PriEsT were developed in an Ubuntu Linux machine using freely available open source tools, as discussed briefly below.

4.7.1 NetBeans IDE and Java

All development has been done using NetBeans 6.9.1 and Java. To create the GUI for the desktop and applet version, the built-in GUI Builder of NetBeans was used. Extra tools, such as the Sun Wireless Toolkit, LWUIT and Android SDK were integrated into NetBeans to have a unique IDE and in this way facilitate the development process.

4.7.2 LWUIT

The Lightweight User Interface Toolkit (LWUIT) is 'a versatile and compact API for creating attractive application user interfaces for mobile devices. LWUIT provides sophisticated Swing-like capabilities without the tremendous power and complexity of Swing.' (Knudsen, 2008). This framework from Oracle allows the creation of mobile user-interfaces that will look the same in any mobile device. The front-end for the mobile version was designed using version 1.5 of this tool, programmed in Java and compiled with the Sun Java Wireless Toolkit.

4.7.3 Sun Java Wireless Toolkit

The Sun Java Wireless Toolkit (Oracle, 2012) is a set of tools for developing J2ME applications based on the Mobile Information Device Profile (MIDP). The toolkit includes the compiler, emulators and performance optimization features. The version 2.5.2 of this toolkit was used to compile, package and test the mobile application.

4.7.4 Android SDK

Android SDK (Google, 2012) is a software development kit that provides a set of libraries, emulators and tools to build, test and debug applications for the

Android OS. Android SDK r20.0.1 was used to compile and package the mobile application to make it available for Android devices.

4.7.5 Retrotranslator

Retrotranslator is a tool that makes Java libraries and applications compatible with previous versions of Java. This tool was necessary to back-port some libraries to make them compatible with J2ME.

4.8 Summary

This chapter has described the re-engineering process that was carried out to prepare PriEsT to fulfil the functional and non-functional requirements. The discussion started with the context and motivation to explain why this process was necessary. Then, the current state of PriEsT was analysed and the limitations of its implementation were highlighted. New requirements were introduced followed by a brief discussion of how the preparation for the new design was addressed. Next, the new design was described and the architecture for the application was presented. The chapter ends discussing the implementation methodology and tools necessary for the development of the project.

The design and implementation details of SA is discussed in the next chapter, and Chapter 6 covers the work carried out to port PriEsT to web and mobile platforms.

The evaluation of the implementation presented in this chapter is discussed in Chapter 7 together with the evaluation of the SA and the platform-specific versions.

Chapter 5
Sensitivity Analysis Implementation

This section covers the details of the design and implementation of three algorithms to perform SA; one-at-a-time, probabilistic simulations and mathematical modelling algorithms. In addition, to show the features of the SA module, a case study is presented. The evaluation and details about the performance of the three SA algorithms are presented in Chapter 7.

5.1 One-at-a-time

5.1.1 Algorithm

This method works by modifying the weight of one element - a criterion or alternative - and then recalculating the ranking of alternatives. The possible range of values for any weight in an AHP model is from 0 to 1, and the sum of weights of the child nodes of any node must be 1. For this reason, when changing the weight of one criterion or alternative, the weights of the other elements must be adjusted proportionally so the weights still sum to 1. Using formula (14) the new weights w' are calculated when a perturbation d is induced on weight w_i.

$$\begin{aligned} w_i' &= w_i + d \\ w_j' &= w_j \left(1 - \frac{d}{\sum_{k \neq i} w_k} \right) \end{aligned} \tag{14}$$

By calculating the priority values P_k of all alternatives A_k when w_i is 0 and 1, two points for each alternative can be obtained: $Q_{1k}=(0, P_{0k})$ and $Q_{2k}=(1, P_{1k})$. Using these two points a line equation is formulated for each alternative which allows the construction of a line chart with the ranking of alternatives as a function of the weight, as shown in Figure 5.1. In addition, by computing the intersection between each pair of lines, the points where rank reversals occur can be found.

This is represented in Algorithm 5.1.

Using this implementation to find the rank reversal points only the equation of line intersection is needed and no further mathematical deduction is necessary, which results in a simpler and easier implementation than using the method (Barker, 2011) discussed in Section 2.7.1, where the formula to calculate the ranking of alternatives has to be expressed as a function of one variable.

Algorithm 5.1. On-at-a-time SA and rank reversals

Calculates rank reversals and ranking of alternatives when the weight w_i changes from 0 to 1.

```
SET w'i=0
CALCULATE the vector of weights W using formula (14)
CALCULATE the ranking of alternatives P0
SET w'i=1
CALCULATE the vector of weights W using formula (14)
CALCULATE the ranking of alternatives P1
FOR EACH k
   SET Q0 = POINT(0, P0k)
   SET Q1 = POINT(0, P1k)
   SET Lk = LINE(Q0, Q1)
END-FOR
SET RANK_REVERSALS = EMTPY LIST
FOR EACH j,k WHERE j!=k
   SET POINT = SEGMENT_INTERSECTION(Lj, Lk)
   IF POINT != NULL THEN
      RANK_REVERSALS.ADD(POINT)
   END-IF
END-FOR
```

5.1.2 Implementation in PriEsT

Following the same design structure used in the re-engineering process, the SA implementation was developed using the MVC architecture. Firstly, the algorithm was implemented and included in the PriEsT library. Secondly, a set of tests were created and executed to ensure the correctness of the results. Lastly, the UI was developed and integrated into the main GUI of PriEsT.

The design of the module allows the user to select the element to analyse. In addition, as PriEsT supports several elicitation methods, the user can select

what elicitation method to use. Once an element and elicitation method is selected, an interactive chart is created where the user can modify the weight of the selected element to obtain the ranking of alternatives with this new value. In addition, the chart presents the current weight for the selected element. This is shown in Figure 5.1.

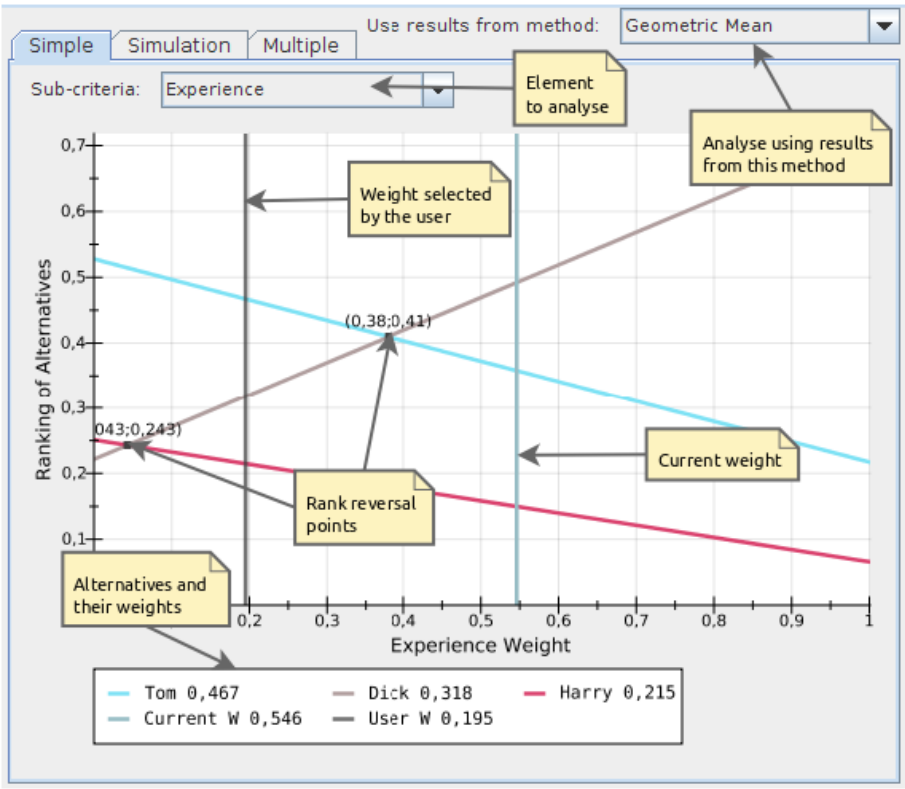


Figure 5.1. On-at-a-time Sensitivity Analysis

From a more technical aspect, this SA method was implemented as a Java class with two important methods: one for obtaining the ranking of alternatives given the weight value of one decision element and one for obtaining the list of all rank reversal points. This class was then used by the GUI to construct and update the chart as the user interacts with it. A detailed class diagram with this design can be found in Appendix A.

5.1.3 Advantages

Some of the advantages of the one-at-a-time method are as follows:

- Simple to implement. There is no great complexity in the implementation, the mathematical formulas are simple and straightforward.
- Simple to use and interpret. The usage of the method is relatively simple, the user only has to select an element from the decision problem and all the rank reversal points are presented.
- Fast response time. The algorithm has no loops of execution hence should be relatively fast. The ranking of alternatives has to be calculated only twice (for weight 0 and 1) and the rank reversal points are calculated using the line-intersection equation.

5.1.4 Limitations

Some of the limitations of the one-at-a-time method are as follows:

- Analysis on one element. This method allows only one element to be analysed. For large problems, with several criteria and alternatives, analysing only one element may not provide enough information for taking a decision. However, the method may be useful if the decision maker is confident about the model and the judgements and there is uncertainty in just a few elements.
- Many alternatives produce a cluttered chart. If the decision problem has a considerable number of alternatives, then the chart may become difficult to read. This may be overcome if the user is allowed to select the list of alternatives to be shown in the chart, unfortunately, this was not implemented because of time constraints and is presented as future work and enhancements.
- Limitation of the implementation. In order to discover all the rank reversal conditions the user has to individually inspect all elements in the problem. All these points could be calculated automatically and presented to the user in the form of a list or table, but for the same reason given above, this has not been implemented.

5.2 Probabilistic Simulation

5.2.1 Algorithm

The algorithm that was implemented is the one proposed by Butler et al. (Butler, 1997) described in Section 2.7.2. This method allows simultaneous changes of all weights in the decision problem to explore the effect on the ranking of alternatives.

The algorithm works by using Monte-Carlo simulations. In every iteration of the algorithm, the weights of the criteria are replaced by random weights and then the ranking of alternatives is calculated and registered. After the simulation is finished, a statistical analysis is performed to find information about the distribution of the ranking of alternatives.

Butler et al. proposed three variations of this method:

a) random weights

b) random weights preserving rank order

c) random weights using a probability distribution

In order to maintain high cohesion in the design, and avoid code duplication, these three variations were collected in one single algorithm, represented in a simplified form in Algorithm 5.2. In this way, the implemented method allows the user to use any distribution with random weights whether or not preserving the original rank order of weights (second variation of the method).

Algorithm 5.2. Probabilistic simulation algorithm.

Performs Monte-Carlo simulations replacing the weights with random values to obtain statistics about the ranking of alternatives.

```
SET T = EMPTY ARRAY
FOR iter = 1 TO ITERATIONS
   FOR EACH c IN CRITERIA
      SET W = RANDOM_GENERATOR.GENERATE_WEIGHTS
      IF PRESERVE_RANK_ORDER == TRUE THEN
         SORT W
      END-IF
```

```
        SET C.WEIGHTS = W
    END-FOR
    CALCULATE the ranking of alternatives P
    SORT P
    FOR i = 1 TO NUMBER_OF_ALTERNATIVES
        SET T_iter,i = P_i
    END-FOR
END-FOR
SET STATS = EMPTY ARRAY
FOR i = 1 TO NUMBER_OF_ALTERNATIVES
    SET STATS_i,Q1 = QUARTILE1(T, i)
    SET STATS_i,Q2 = QUARTILE2(T, i)
    SET STATS_i,Q3 = QUARTILE3(T, i)
    SET STATS_i,MIN = MIN(T, i)
    SET STATS_i,MAX = MAX(T, i)
    SET STATS_i,MEDIAN = MEDIAN(T, i)
    SET STATS_i,MEAN = MEAN(T, i)
END-FOR
```

In each iteration, the algorithm uses a random number generator to generate the random weights, in this way the algorithm can generate random weights (first variation of the method) if the random generator is the uniform random number generator, or can generate weights from a probability distribution (third variation of the method) if the random generator is a custom number generator. Three random number generators were implemented in PriEsT according to the uniform, gamma and triangular probability distributions. The density functions of the last two are depicted in Figure 5.2.

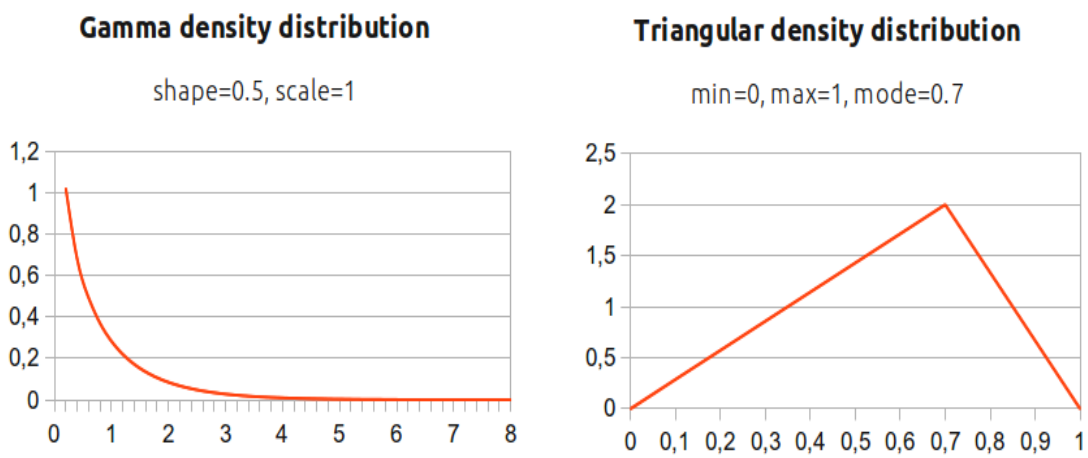


Figure 5.2. Gamma and Triangular Distribution

The output of the algorithm is a set of statistical measures of the behaviour of each alternative. These measures are described in Table 5.1.

Measure	*Description*
Min	The minimum or best ranking obtained by the alternative
Max	The maximum or worst ranking obtained by the alternative
Quartile 1	The alternative obtained this ranking or better in 25% of the iterations
Quartile 2	The alternative obtained this ranking or better in 50% of the iterations
Quartile 3	The alternative obtained this ranking or better in 75% of the iterations
Median	The median ranking obtained by the alternative
Mean	The mean ranking obtained by the alternative

Table 5.1. Statistical Measures in Probabilistic Simulation SA

5.2.1.1 Uniform weight generator

This method generates weights completely at random ignoring the values of the current weights. The generated values are in the range from 0 to 1 and sum to 1. This method is represented in Algorithm 5.3.

Algorithm 5.3. Uniform weight generator

Generates N random weights

```
SET R = ARRAY[N-1]
FOR i = 1 to N-1
   SET Ri = RANDOM BETWEEN 0 AND 1
END-FOR
SORT R
SET WEIGHTS = ARRAY[N]
FOR i = 2 to N
   SET WEIGHTSi = Ri - Ri-1
END-FOR
SET WEIGHTSN = 1 - RN-2
```

5.2.1.2 Gamma weight generator

This generator produces random weights from the gamma distribution using the current weight as the shape parameter and a constant value as the scale parameter to the distribution. The generated values are then normalised, each value is divided by their sum to make them sum to 1. The algorithm used for generating random gamma values has been extracted from the MALLET open source library developed by McCallum (McCallum, 2002).

5.2.1.3 Triangular weight generator

Although Butler at al. only mention the uniform and gamma distributions, the triangular distribution was also implemented to give the user an additional option to analyse the problem. The triangular distribution takes 3 parameters minimum, maximum and mode. The minimum is 0, maximum is 1 and the current weight is used as the mode. In this way, the generated weights will tend to be close to the current weight, but still can be distributed through the whole domain of possible values.

The advantage of this distribution, is that it generates all possible random weights taking into consideration the current weight, hence, values that are close to the current weight are more likely to be generated. In addition, the implementation is simple and the execution is fast as there are no loops or costly operations (only multiplications and additions).

The method to generate a random weight from this distribution is shown in Algorithm 5.4.

Algorithm 5.4. Triangular weight generator

Generates N random weights from the triangular distribution

```
SET U = RANDOM BETWEEN 0 AND 1
SET F = (mode - min) / (max - min)
IF U < F THEN
   SET W = min + SQRT(U*(max - min)*(mode - min))
ELSE
   SET W = max - SQRT((1 - U)*(max - min)*(max - mode))
END-IF
```

5.2.2 Implementation in PriEsT

In the same way as with the previous SA method, this method was implemented independently from the UI. Once the method was developed and tested in the library, the GUI was created and added to the PriEsT application.

The method proposed by Butler et al. analyses all criteria that are non-leaf nodes in the hierarchy tree. In other words, the weights of each element in the decision problem are replaced by random weights with the exception of the weights of alternatives. In PriEsT this method was implemented in a more flexible way allowing the user to select which decision elements to include in the analysis. By default, all non-leaf nodes are selected, but the user is free to change this selection. The inclusion of a decision element in the simulation means that the weight of its child nodes will be replaced by random weights.

Although the underlying implementation in the PriEsT library allows the number of iterations performed in the simulation to change , to keep the UI simple the number of iterations has been fixed to 10000 which is twice as many as Butler et al. used in their research.

As shown in Figure 5.3, the user can enable or disable the preservation of the original rank order, and can select both the probability distribution and the nodes to include in the simulation. The output of the algorithm is presented to the user as a box-chart with 5 elements: best ranking, worst ranking, mean ranking, quartile 1 and quartile 3. The green bar in the snapshot indicates that in 50% of the results of the simulation the alternative “Tech 5” got a ranking between 2 and 3, and a mean ranking around 2.8. The chart also shows that alternative “Tech 2” is the best option, with a mean ranking of 1.4 and being in the 3rd position as the worst case.

In order to be able to include more probability distributions in the PriEsT library and therefore in the application without modifying existing code, the design of this SA method was decoupled from the random weights generators. In this way, a new weight generator can be developed and included in the list of available distributions without modifying the main algorithm. To do so, the class implementing the new weight generator only needs to be referenced in the

appropriate factory object. A detailed class diagram with this design can be found in Appendix A.

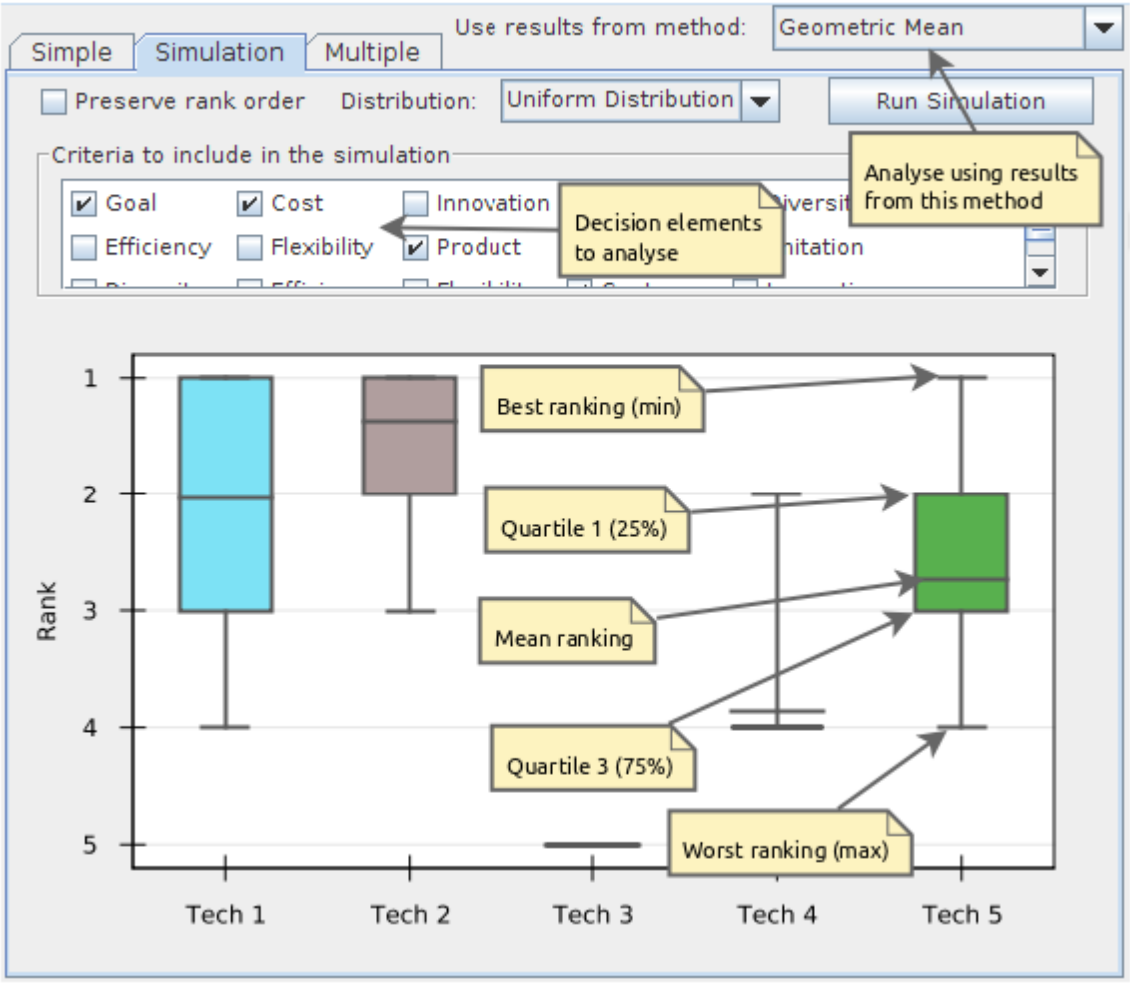


Figure 5.3. Probabilistic Simulation Sensitivity Analysis

5.2.3 Advantages

Some of the advantages of the probabilistic simulation method are as follows:

- All decision elements are considered. This method allows simulation to be carried out using all the decision elements in the model. The user can select any number of criteria from any level of the hierarchy tree. This feature enables the decision maker to gain a very useful insight into the decision problem at hand by performing several simulations using different criteria and probability distributions.

- Simplifies the decision problem. The simulation method may be used to simplify the model by filtering out the alternatives that are consistently performing the lowest. If the simulation's results indicate that one or more alternatives are never top ranked regardless of the relative weights of the criteria, then the decision maker can remove them from the model and focus only on alternatives that are superior.
- Simple and easy to understand. The method is relatively easy to use. The user has to select the distribution, choose whether or not to preserve the rank order and select the criteria. The box-chart presents the results of the simulation in a straightforward format that allows easy identification of the alternatives and their performance.
- Flexible Implementation. The algorithm has been implemented in a flexible way that allows the development and inclusion of more probability distributions in a simple way. By extending the method with more distributions more heterogeneous problems may be analysed as new distributions may reflect more accurately other situations such as group decision making or different types of criteria.

5.2.4 Limitations

Some of the limitations of the probabilistic simulation method are as follows:

- Performance. Although is not really a limitation or problem, the time required to run the simulation is considerably longer than the previous SA method, albeit it is only a few seconds in total. For example, for a problem with 25 criteria, the simulation takes around 2 seconds with a 1.6Ghz processor. It is worth noting that the modern implementations of the JVM contains a built-in Just In Time (JIT) compiler. The JIT analyses the execution of the application while it runs and optimises it by re-compiling the Java byte-code to machine code. The advantage of this is that the execution of the simulation for the first time causes the JIT to optimise the code so in further executions the performance is greatly increased. More details about performance can be found in Section 7.2.

- No details. The probabilistic approach generates aggregated results which are presented as a box-chart. Hence, the user does not know what combination of weights produced the results that are being displayed. These details could be registered as the simulation runs, but it would constitute a great amount of data as for each iteration there is a different weight combination and it would be too complex to display it in a way that it can be useful and readable for the user. It may be possible to record the data, provide it in summary details to the user and allow them to drill-down into the details should they wish.

5.3 Mathematical Modelling

5.3.1 Algorithm

The aim of this method is to explore the impact of single or multiple changes to the weights of the decision elements in any level of the hierarchy tree. Chen and Kocaoglu (Chen, 2008) proposed a set of theorems and formulas to analyse the allowable range of perturbations that may be introduced into the model without causing a rank reversal. Their approach can be used to obtain the allowable range of perturbations on any number of decision elements given that they are in the same level of the hierarchy. If a perturbation P_i is induced on weight w_i, then the new weight $w'_i = w_i + P_i$.

Depending on the level where the perturbations are induced in the hierarchy, different formulas are used to calculate the allowable range, which is expressed as a system of inequalities. Figure 5.4 represents a general decision problem shown as a hierarchy tree. The criteria in the top level are child nodes of the root or main goal of the model. Middle level criteria correspond to nodes that have a criterion node as a parent and may have criteria or alternatives as children. The alternatives are located in the bottom level.

The algorithms to find the allowable range of perturbations in each of the three levels when N perturbations are induced on N of the elements e_i with weights w_i, $i=1...N$ are given next.

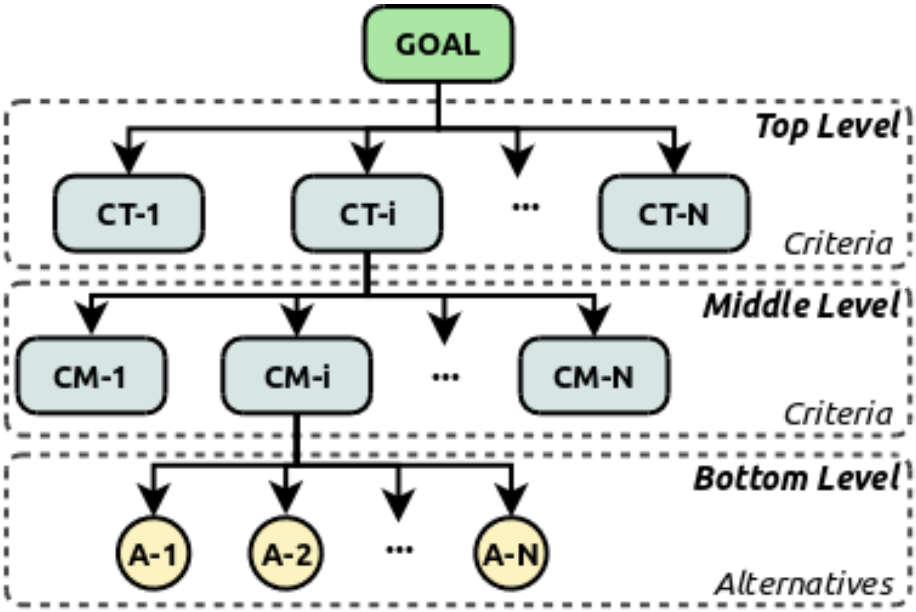


Figure 5.4. Hierarchy Tree for Mathematical SA

CT: Criteria in Top Level, CM: Criteria in Middle Level

5.3.1.1 Top Level Algorithm

This algorithm is based on Theorem 1 from (Chen, 2008). A simplified version of the theorem is as follows:

Let T_{l*m} denote M perturbations induced on M of the L criteria C_l, which are C_{l*m}; the original ranking of A_r and A_{r+n} will not reverse if :

$$\lambda \geq T_{l*1} * \lambda_{l1} + T_{l*2} * \lambda_{l2} + \ldots + T_{l*M} * \lambda_{lM}$$

where $\lambda = P_r - P_{r+n}$,

$$\lambda_{lm} = P_{r+n,l*m} - P_{r,l*m} - \sum_{i=1, i \neq l*m}^{L} \left[P_{r+n,l} * \frac{w_{Ci}}{\sum_{j \neq l*m}^{L} w_{Cj}} + P_{r,l} * \frac{w_{Ci}}{\sum_{j \neq l*m}^{L} w_{Cj}} \right]$$

P_i: global priority of alternative i

$P_{i,j}$: local priority of alternative i to criterion j

The top-ranked alternative will remain at the top rank if the above condition is satisfied for all $r = 1$ and $n = 1,2 \ldots I-1$.

The rank order of all A_i's will remain unchanged if the above condition is satisfied for all $r = 1,2, I-1$, and $n = 1$.

The implemented algorithm is as follows:

Algorithm 5.5 Allowable region for top level criteria

Calculates the allowable region of perturbations where alternative A_k is the most preferred.

P: represents the global priority values of the alternatives.

E: represents the set of N elements that are being perturbed.

```
FOR i = 1 to NUMBER_OF_ALTERNATIVES AND i != k
   SET sum = 0
   FOR j = 1 to N
      IF ej IS_NOT_IN E THEN
         SET PJ = RANKING of alternatives in criterion ej
         SET sum = sum + (PJk - PJi) * wj
      END-IF
   END-FOR
   SET S = sum / SUM(w)
   FOR j = 1 to N
      SET PJ = RANKING of alternatives in criterion ej
      SET ci,j = PJj - PJk + S
      SET Di = Pk - Pi
   END-FOR
END-FOR
```

5.3.1.2 Middle Level Algorithm

This algorithm is based on Theorem 2, Corollary 2.2 from (Chen, 2008). A simplified version of the theorem is as follows:

Let T_{k*ml*} denote M perturbations induced on M of the K criteria C_{kl}, which are C_{k*ml*} (sub-criteria C_{k*} of criterion C_{l*});
the original ranking of A_r and A_{r+n} will not reverse if :

$$\lambda \geq T_{k*1l*}*\lambda_{k_1 l}+T_{k*2l*}*\lambda_{k_2 l}+...+T_{k*_M l*}*\lambda_{k_M l}$$

where $\lambda = P_r - P_{r+n}$,

$$\lambda_{km\,l} = w_{Cl*}*\left[P_{r+n,k*m} - P_{r,k*m} + \sum_{i=i,i\neq k*m}^{K}(P_{r,i}-P_{r+n,i})*\frac{w_{Ck\,l*}}{\sum_{j\neq l*m}^{K} w_{Cj\,l*}}\right]$$

The top-ranked alternative will remain at the top rank if the above condition is satisfied for all $r=1$ and $n=1,2...I-1$.
The rank order of all A_i's will remain unchanged if the above condition is satisfied for all $r=1,2,I-1$, and $n=1$.

The implemented algorithm is as follows:

Algorithm 5.6 Allowable region for middle level criteria

Calculates the allowable region of perturbations where alternative A_k is the most preferred.

P: represents the global priority values of the alternatives.

E: represents the set of N elements that are being perturbed.

```
FOR i = 1 to NUMBER_OF_ALTERNATIVES AND i != k
   SET sum = 0
   FOR j = 1 to N
      IF ej IS_NOT_IN E THEN
         SET PJ = RANKING of alternatives in criterion ej
         SET sum = sum + (PJk - PJi) * wj
      END-IF
   END-FOR
   SET S = sum / SUM(w)
   FOR j = 1 to N
      SET PJ = RANKING of alternatives in criterion ej
      SET CJ = CONTRIBUTION of parent of ej to the goal
      SET ci,j =  CJ * (PJj - PJk + S)
      SET Di = Pk - Pi
   END-FOR
END-FOR
```

5.3.1.3 Bottom Level Algorithm

This algorithm is based on Theorem 3, Corollary 3.2 from (Chen, 2008). The theorem is as follows:

Let T_{i*mj*} denote M perturbations induced on M of the I alternatives A_{ij}, which are A_{i*mj*}(alternative A_{i*} under criterion C_{j*});
the original ranking of A_r and A_{r+n} will not reverse if :

$$P_r - P_{r+n} \geq \sum_{m=1}^{M} C_{j*} * T_{i*m,j*} * \frac{P_{r,j*} - P_{r+n,j*}}{\sum_{i=1, i \neq i*}^{I} P_{i,j*}}$$

(when perturbations are induced on neither $A_{r,j}$ nor $A_{r+n,j}$)

$$P_{r*} - P_{(r+n)*} \geq \left(T_{(r+n)*m,j*} - T_{r*m,j*}\right) * C_{j*}$$

(when perturbations are induced on both $A_{r,j}$ and $A_{r+n,j}$)

$$P_r - P_{(r+n)*} \geq \sum_{\substack{m=1 \\ i \neq r+n}}^{M} \left[T_{i*m,j*} * C_{j*} * \left[\frac{P_{r,j*}}{\sum_{i=1, i \neq i*}^{I} P_{i,j*}} \right] + T_{(r+n)*m,j*} * C_{j*} * \left[1 + \frac{P_{r,j*}}{\sum_{i=1, i \neq i*}^{I} P_{i,j*}} \right] \right]$$

(if one of the perturbations is induced on $A_{r+n,j}$)

$$P_{r*} - P_{r+n*} \geq -\sum_{\substack{m=1 \\ i \neq r}}^{M} \left[T_{i*m,j*} * C_{j*} * \left[\frac{P_{r+n,j*}}{\sum_{i=1, i \neq i*}^{I} P_{i,j*}} \right] - T_{r*m,j*} * C_{j*} * \left[1 + \frac{P_{r+n,j*}}{\sum_{i=1, i \neq i*}^{I} P_{i,j*}} \right] \right]$$

(if one of the perturbations is induced on $A_{r,j}$)

C_{j*}: Contribution of criterion C_j to the overall goal

The top-ranked alternative will remain at the top rank if the above condition is satisfied for all $r=1$ and $n=1,2...I-1$.
The rank order of all A_i's will remain unchanged if the above condition is satisfied for all $r=1,2, I-1$, and $n=1$.

The implemented algorithm is as follows:

Algorithm 5.7 Allowable region for bottom level alternatives

Calculates the allowable region of perturbations where alternative A_k is the most preferred.

```
FOR i = 1 to NUMBER_OF_ALTERNATIVES AND i != k
   SET sum = 1 - SUM(w)
   FOR j = 1 to N
      SET PJ = RANKING of alternatives in criterion ej
      SET CJ = CONTRIBUTION of parent of ej to the goal
      IF ei IS_NOT_IN E AND ek IS_NOT_IN E THEN
         SET ci,j =  CJ * (wk - wi) / sum
      ELSE IF ei IS_IN E AND ek IS_IN E THEN
         IF k == j THEN
            SET ci,j =  -CJ
         ELSE
            SET ci,j =  CJ
         END-IF
      ELSE IF ei IS_IN E AND ek IS_NOT_IN E THEN
         IF i != j THEN
            SET ci,j =  CJ * wk / sum
         ELSE
            SET ci,j =  CJ * ( 1 + wk ) / sum
         END-IF
      ELSE
         IF k != j THEN
            SET ci,j =  -CJ * wi / sum
         ELSE
            SET ci,j =  CJ * ( 1 + wi ) / sum
         END-IF
      END-IF
      SET Di = Pk - Pi
   END-FOR
END-FOR
```

The application of each of these algorithms produces a system of inequalities similar to the one shown in formula (15) that represents the allowable range of perturbations for an alternative to be the most preferred.

$$\begin{gathered} c_{1,1}P_1+\ldots+c_{1,i}P_i+\ldots+c_{1,N}P_N \le D_1 \\ c_{2,1}P_1+\ldots+c_{2,i}P_i+\ldots+c_{2,N}P_N \le D_2 \\ \vdots \\ c_{N,1}P_1+\ldots+c_{N,i}P_i+\ldots+c_{N,N}P_N \le D_N \end{gathered} \tag{15}$$

If one perturbation is induced, then the solution to the system of inequalities will represent a domain in one dimension. For two perturbations a two-dimensional domain is generated, for three perturbations a three-dimensional domain is generated, and so on. Figure 5.5 shows an example of two-dimensional and three-dimensional allowable regions where an alternative will be top ranked as long as the perturbations lie within the region.

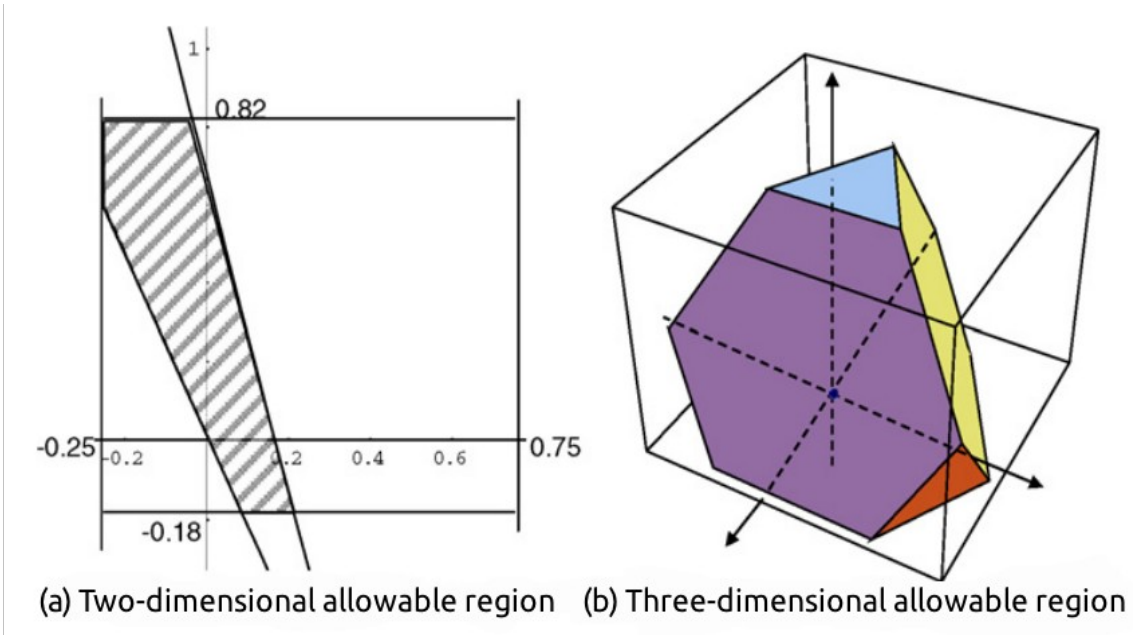


Figure 5.5. Allowable range of perturbations (Chen, 2008)

In addition to the inequalities produced by algorithms 5.5, 5.6 and 5.7, a set of inequalities representing the domain constraints must be considered. If a perturbation P_i is induced on weight w_i, then P_i must satisfy formula (16):

$$\begin{aligned} &-w_i \le P_i \le 1 - w_i \\ &\textit{it can be expressed as}: \\ &-P_i \le w_i \\ &\quad P_i \le 1 - w_i \end{aligned} \tag{16}$$

Additionally, as the weights must sum to 1, the perturbations also must satisfy:

$$-\sum w_i \leq \sum P_i \leq 1 - \sum w_i$$

$$\begin{aligned} &it\ can\ be\ expressed\ as: \\ &-\sum P_i \leq \sum w_i \\ &\sum P_i \leq 1 - \sum w_i \end{aligned} \tag{17}$$

By executing the algorithm for each alternative and including the domain inequalities (16) and (17), the region where each alternative is the most preferred one can be identified by solving the system of inequalities. If one alternative is never the most preferred, then the system of inequalities will have no solution and the allowable region for this alternative will be empty.

5.3.1.4 Sensitivity Coefficients

In addition to the allowable region, two sensitivity coefficients can be calculated (Chen, 2008).

- OPSC: Operating point sensitivity coefficient. It is defined as the shortest distance from the current weights to the edge of the allowable region. It represents the minimum change that must be induced on the weights to cause a rank reversal.
- TSC: Total sensitivity coefficient. It is defined as the ratio of the size (length, area, volume or hyper-volume depending on the number of perturbations) of the allowable region of the current top alternative in relation to the total size of the weight space. This coefficient specifies that the shorter the allowable region is, the more sensitive the decision problem is to variations of the weights.

5.3.2 Implementation in PriEsT

Following the general architecture of PriEsT, this SA method was implemented first in the PriEsT library and then a front-end was designed, developed and integrated into the main GUI of the application. This design allows the reuse of this and the previous SA algorithms using different front-ends, as will be

discussed in the next chapter.

To make this SA method easy to use and interpret by the user, in addition to the system of inequalities that represent the allowable region of perturbations for each alternative, a polygon defining the boundaries of each region is also calculated so that it can be displayed in the application. As an allowable region will be n-dimensional when n perturbations are induced, to construct a two-dimensional polygon, the n-dimensional allowable region is transformed or projected into a 2-dimensional plane. Having a two-dimensional representation of the allowable region facilitates the process of constructing a chart and more importantly, of computing the area and shortest distance to the edges in order to calculate the OPSC and TSC coefficients. In addition, this process replaces the perturbation variables for the weight variables, so the resulting polygon is easier to interpret.

The transformation is separated into four categories according to dimensionality of the allowable region: uni-dimensional, two-dimensional, three-dimensional and multi-dimensional.

5.3.2.1 Uni-dimensional Transformation

The system of inequalities (see Formula (15)) together with the domain inequalities (Formulas (16) and (17)) that defines the allowable region contains one variable, the perturbation P_i induced on the weight w_i. Solving this inequalities and replacing $P_i = w'_i - w_i$ will result in:

$$\begin{aligned} w'_i &\geq a \\ w'_i &\leq b \end{aligned} \tag{18}$$

To convert this result into a two-dimensional polygon, the variable w'_i is mapped on the x-axis and values for the y-axis are added in a way that the area of all the regions will sum to 1. As a consequence, the polygon will have five points: *(a,0)*, *(a,1)*, *(b,1)*, *(b,0)* and *(a,0)*. The first point is repeated to close the polygon. An example of the resulting polygons is shown in Figure 5.6.

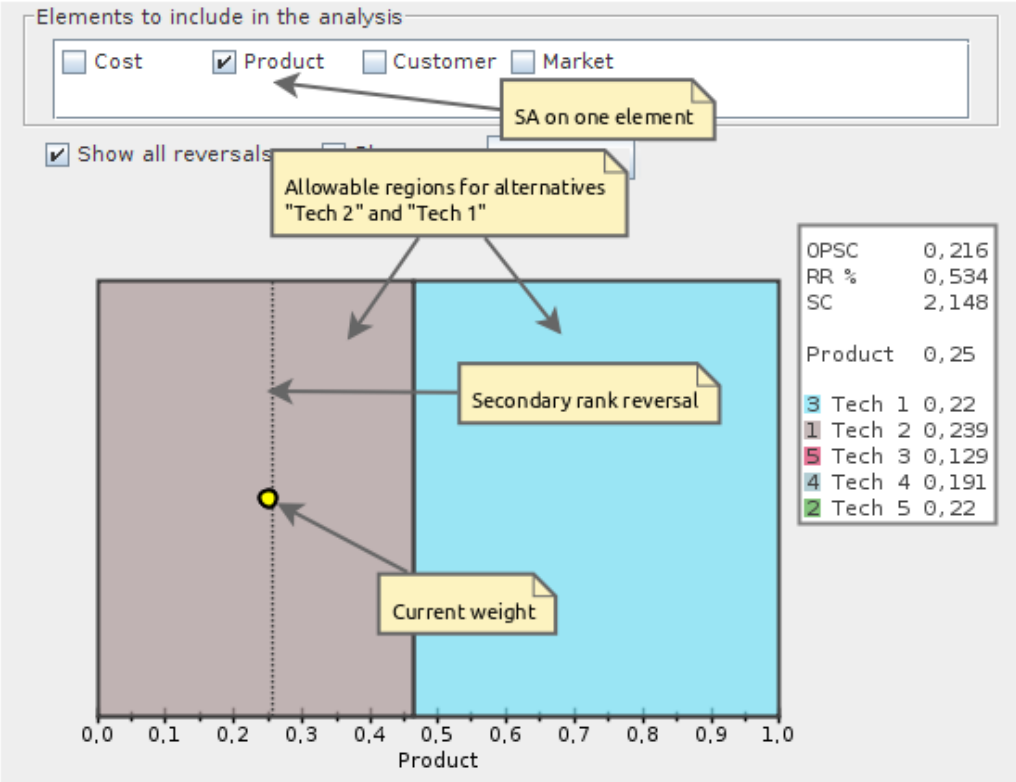


Figure 5.6. Uni-dimensional Projection

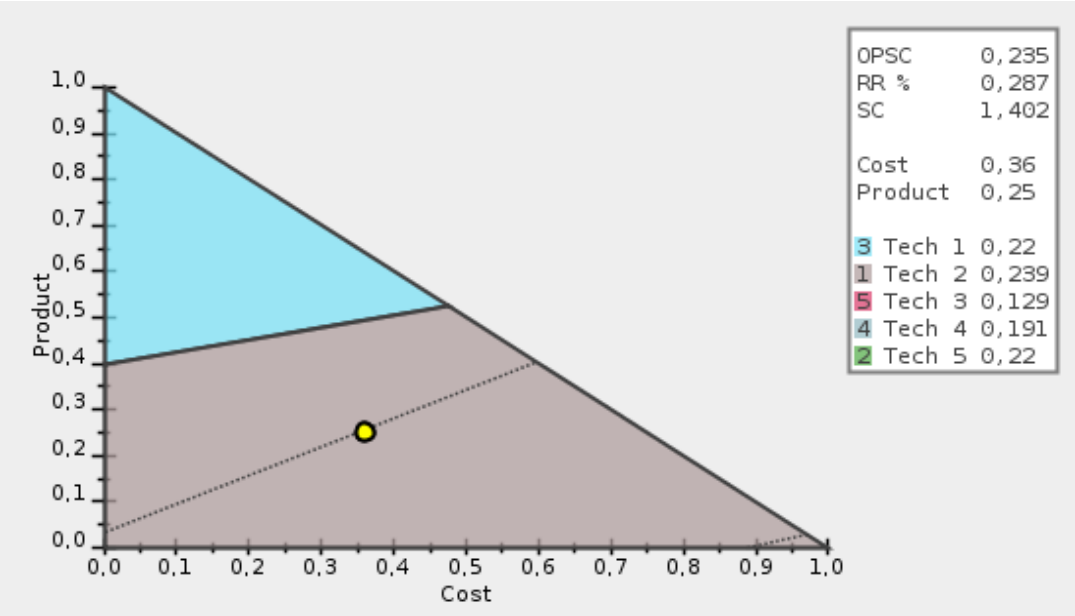


Figure 5.7. Two-dimensional Projection

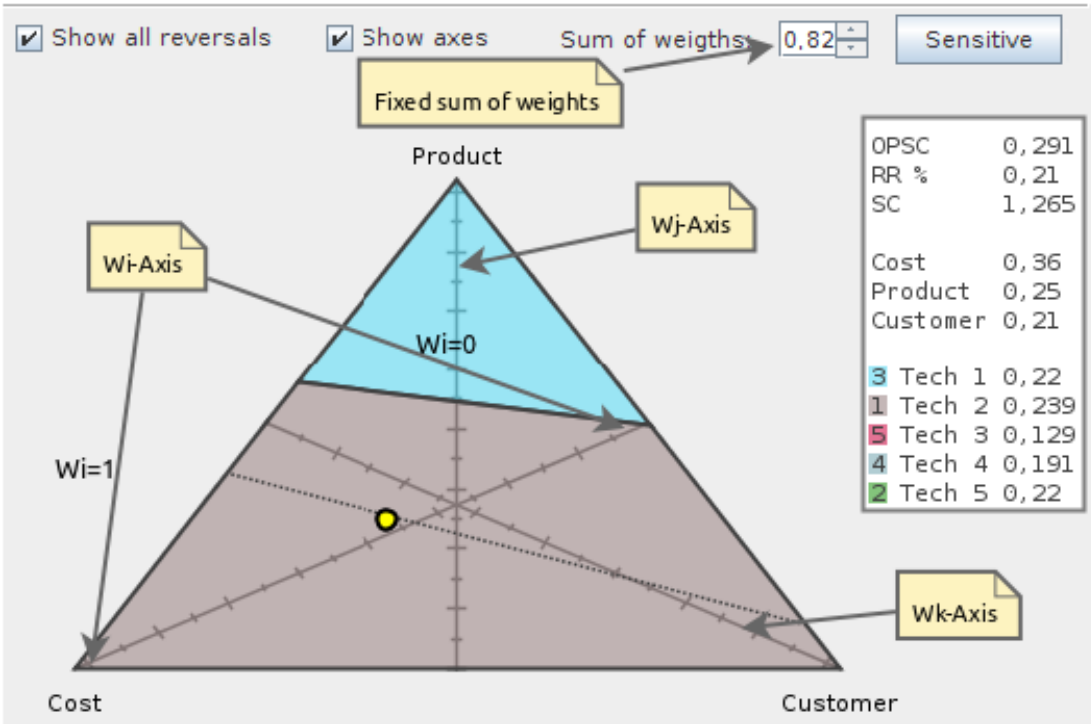


Figure 5.8. Three-Dimensional Projection

5.3.2.2 Two-dimensional Transformation

In this case, as there are two variables in the system of inequalities, no transformation is required. The whole domain of possible values corresponds to a triangle-shaped region with one variable on the x-axis and the other on the y-axis, as shown in Figure 5.7.

5.3.2.3 Three-dimensional Transformation

In order to project the three-dimensional allowable region to a two-dimensional plane the domain of possible weights must be restricted by converting the inequality $w_i+w_j+w_k<=1$ to the equality $w_i+w_j+w_k=s$, where s can take any value between 0 and 1. By changing the value of s the whole domain of possible values can still be inspected.

As the sensitivity coefficients OPSC and TSC depend on the distance from the current weight to the edges and the area of the region, the projection into the two-dimensional plane must not perturb distances nor distort the original

shape of the region. Erkut and Tarimcilar (Erkut, 1991) proposed a method for finding the boundaries of each region and projecting the three-dimensional region into a two-dimensional plane with no distortion of images. This method allows the weights to be converted from a three-dimensional point (w_i, w_j, w_k) into a two-dimensional point (x,y) and vice-versa.

As shown in Figure 5.8, the user is allowed to change the value of s (sum of weights) to inspect the sensitivity of the model for all possible combinations of the three weights. In each corner of the triangle, one of the weights has a value of s and the others 0 and in the middle of the base, one of the weights is 0 and the other two are $s/2$.

5.3.2.4 Multi-dimensional Transformation

This transformation was not implemented because further research is needed to investigate how a multi-dimensional polytope with 4 or more dimensions can be represented in a two-dimensional plane. Because of time constraints, a research on this matter is beyond the scope of the project and will be presented as future work in Chapter 8.

5.3.2.5 Sensitivity Coefficients and Rank Reversal

The implemented SA module presents three coefficients to the user: OPSC, RR % (Rank Reversal Probability) and SC (Sensitivity Coefficient), as shown in Figures 5.6, 5.7 and 5.8.

As TSC represents the ratio of the area of the top alternative's allowable region to the total area, then 1-TSC will represent the probability that a rank reversal occurs for any combination of weights. That is, if TSC is close to 0, then 1-TSC will be close to 1 and vice-versa. 1-TSC was named Rank Reversal Probability and is presented as RR%.

In a similar way, $SC=1/TSC$ is more meaningful, as a small value of SC will indicate a low sensitivity to changes in the weight while a large value of SC will indicate that the problem is very sensitive to changes to the relative importance of criteria.

Additionally, if a rank reversal occurs for some combination of weights but it does not change the top alternative, it is included in the charts as a dotted line. Secondary rank reversals are displayed in Figures 5.6, 5.7 and 5.8.

5.3.2.6 Most Sensitive Decision Element

The element (or set of elements as up to three elements may be selected) with the lowest OPSC and TSC can be considered as the most sensitive decision element. If the lowest OPSC and TSC belong to different elements, then the element with the lowest *OPSC * TSC* can be considered to be the most sensitive.

In order to locate this element, an SA is performed at each node of the decision problem and the OPSC and TSC values are recorded. The analysis is done using all possible combinations of the child nodes in groups of one, two and three elements. The total number of different combinations for a node with *n* child elements is given in formula (19):

$$combinations = n + \frac{n!}{(n-2)!\,2!} + \frac{n!}{(n-3)!\,3!} \tag{19}$$

Once all nodes are analysed, two decision elements are identified: the element with the lowest OPSC and the one that obtained the lowest TSC. If these are different elements, then the element with the lowest *OPSC * TSC* is considered as the most sensitive element of the decision problem.

5.3.3 Advantages

Some of the advantages of the mathematical modelling method are as follows:

- Simple to use and interpret. Usage of the SA method is relatively simple, the user only has to select from one to three elements from the decision problem and the allowable regions are plotted automatically presenting the sensitivity of the decision elements in a visual way . The chart is interactive as it allows the user to select any point inside the polygon to obtain the associated ranking of alternatives.

- **Fast response time.** The algorithm has no execution loops hence it has relatively fast execution. The chart is updated instantly after the user selects the decision elements.
- **Sensitivity coefficients:** Three sensitivity coefficients (OPSC, RR% and SC) and the most sensitive decision element are presented to the user.

5.3.4 Limitations

Some of the limitations of the mathematical modelling method are as follows:

- **Chart available for up to three elements**. This method allows any number of decision elements to be analysed, however, the visualisation of the results in a chart is limited to three elements. More research is necessary to find a method for displaying results when more than three elements are considered.

5.4 Select a Single Solution from PrInT

The optimisation-based method PrInT (Siraj, 2011) generates more than one non-dominated solution from a PC matrix. Automated selection of a single solution was suggested as an area for further work by Siraj (Siraj, 2011) and is widely regarded as a major challenge in the area. By using the implemented mathematical SA method a single solution can be selected from the set of non-dominated solutions by choosing the solution that minimises the sensitivity of the final ranking of alternatives.

A decision problem may have any number of decision elements and for each element PrInT produces a set of solutions. For each solution an SA is conducted to find the most sensitive element and its associated OPSC value. The solution that generates the most sensitive element with the highest OPSC value can be selected as the best solution for that decision element, as that solution produces the most robust final ranking of alternatives.

This novel method has been implemented in PriEsT as represented in Algorithm

5.8 and can be used for any elicitation method that generates more than one solution (e.g. TOP (Mikhailov, 2006) and PrInT).

Algorithm 5.8 Select a single solution from PrInT

```
FOR EACH decision element E in the problem
   SET MAX = 0
   SET SOLUTIONE = S0
   FOR EACH solution Si generated by PrInT for E
      SET SE = find most sensitive element using Si
      SET OPSC = SE.OPSC
      IF OPSC > MAX THEN
         MAX = OPSC
         SOLUTIONE = Si
      END-IF
   END-FOR
END-FOR
```

To illustrate the method, consider the example given in Section 2.6. If the PrInT method is used to obtain the weights for the first level criteria from the PCM given in Table 2.2, then 176 non-dominated solutions are generated. Table 5.2 shows some of these solutions.

Solution	*WPRICE*	*WMPG*	*WAmenities*	*OPSC*	*RR%*
1	0.594	0.093	0.313	0.1254	66.7%
2	0.596	0.094	0.310	0.1277	66.7%
56	0.702	0.067	0.231	0.2104	66.7%
127	0.715	0.062	0.223	0.2199	66.7%
128	0.647	0.083	0.271	0.1681	66.7%
129	0.646	0.082	0.272	0.1673	66.7%
158	0.664	0.077	0.259	0.1809	66.7%
175	0.680	0.075	0.245	0.1943	66.7%
176	0.674	0.074	0.252	0.1890	66.7%

Table 5.2. PrInT Solutions

The minimum OPSC is 0.1254 and is found in Solution 1, while the maximum is

0.2199 and is found in solution 127. By choosing solution 127, the minimum change of weights necessary to cause a rank reversal is 0.2199 which represents a 75.4% more robust ranking than solution 1.

By using this approach for selecting a single solution from elicitation methods that generate more than one, the final ranking of alternatives is more robust to changes in criteria weights giving the DM more confidence about the suggested ranking.

5.5 Case Study: Supplier Selection Problem

A recent study (Bruno, 2012) that focuses on the supplier selection problem (SSP) will be used to demonstrate the features of the three SA methods. The study uses AHP to construct a model to evaluate suppliers in order to improve a firm's competitiveness.

The model contains two levels of criteria. The first level includes process and product quality (PPQ), service level (SL), management and innovation (MI), and financial position (FP). The weights for these four criteria are:

$$w_{PPQ} = 41.7\%,\ w_{SL} = 35.9\%,\ w_{MI} = 9.5\%,\ w_{FP} = 13\%$$

Each of these criteria contains three sub-criteria, as shown in Figure 5.9. There are four supplier companies under consideration: alpha, beta, gamma and delta. The final weights calculated using either the EV or GM method are:

$$w_{alpha} = 24.6\%,\ w_{beta} = 29.8\%,\ w_{gamma} = 21.3\%,\ w_{delta} = 24.2\%$$

The most preferred alternative is beta with a weight of 29.8%, followed by alpha and delta with a weight of 24.6% and 24.2% respectively.

5.5.1 Sensitivity Analysis using PriEsT

The decision problem is first analysed using the probabilistic simulation approach. By using the random generator the likely rank order of the alternatives may be identified independently from criteria weights.

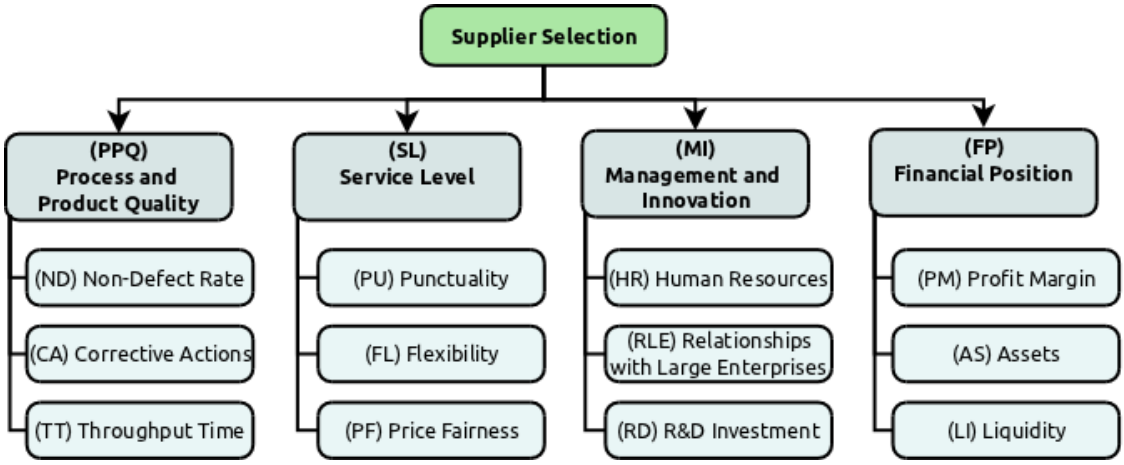
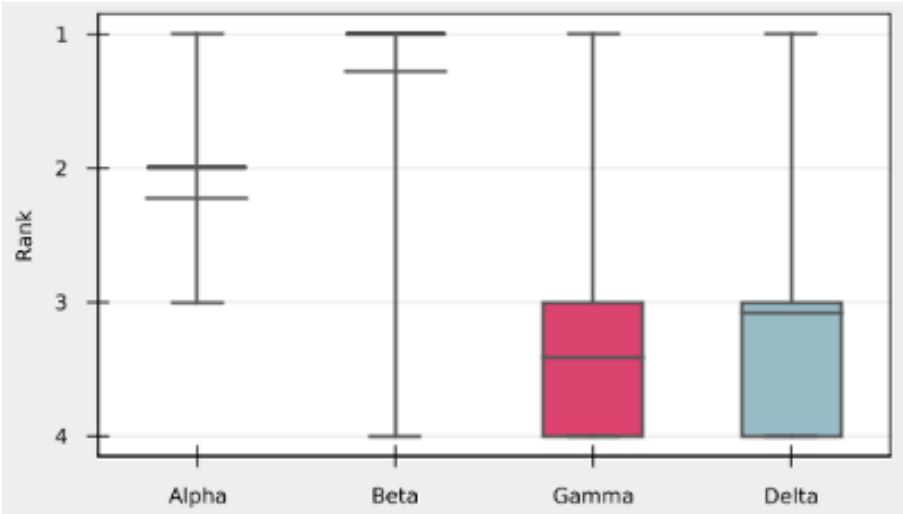


Figure 5.9. AHP model for supplier selection

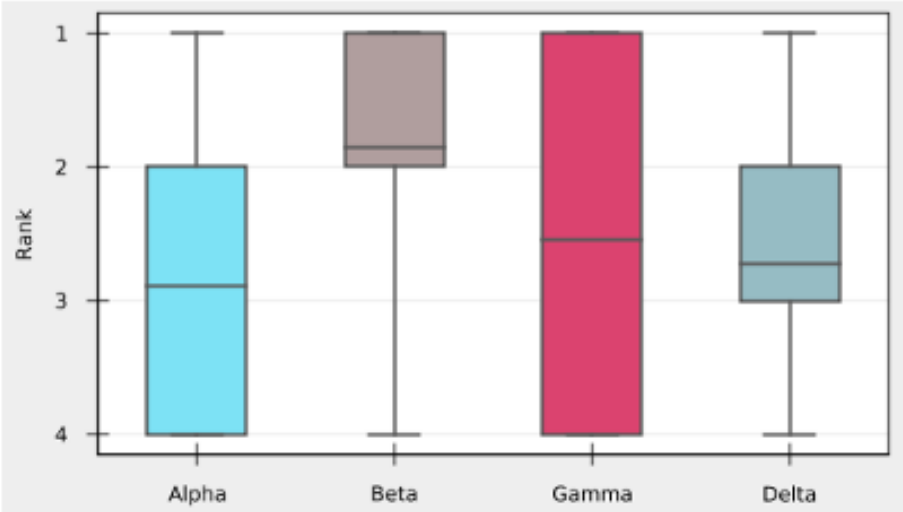
Figure 5.10a shows the ranking of alternatives when uniformly distributed random weights are used for the top-level criteria (PPQ, SL, MI and FP). Supplier beta has a mean ranking around 1.25 with the best ranking of 1 and the worst as 4. In the 50% of the simulation iterations (quartile1 and quartile3), beta was top ranked. Supplier alpha had a mean ranking of 2.2, with best and worst rankings of 1 and 3 respectively. Although gamma and delta are ranked between 3rd and 4th half of the time, for some combination of weights they become the top ranked alternative.

The simulation was executed again replacing all criteria weights with random weights, as shown in Figure 5.10b. This time beta is still the best alternative, however gamma, which was the worst alternative with 21.3%, now is in second place with a mean ranking of 2.5 and a ranking between 1 and 4 for 50% of the iterations. Similar results are obtained using the gamma or triangular distribution.

Given this scenario each alternative has to be considered in the further analysis as each may be the most preferred for some combination of weights.



(a) Simulation using random weights for top-level



(b) Simulation using random weights for all criteria

Figure 5.10. Ranking of alternatives using probabilistic simulation

To discover what weights cause the ranking to change, the one-at-a-time and mathematical analyses are performed. The one-at-a-time method shows that a decrease in the weight of PPQ from 41.6% to 12.2% causes gamma to be the preferred alternative with a priority of 25.8%, as shown in Figure 5.11. Similarly, for weights of SL in the interval 53.8% to 55.4% alpha is the best alternative, and from 55.4% to 100% gamma is the most preferred supplier.

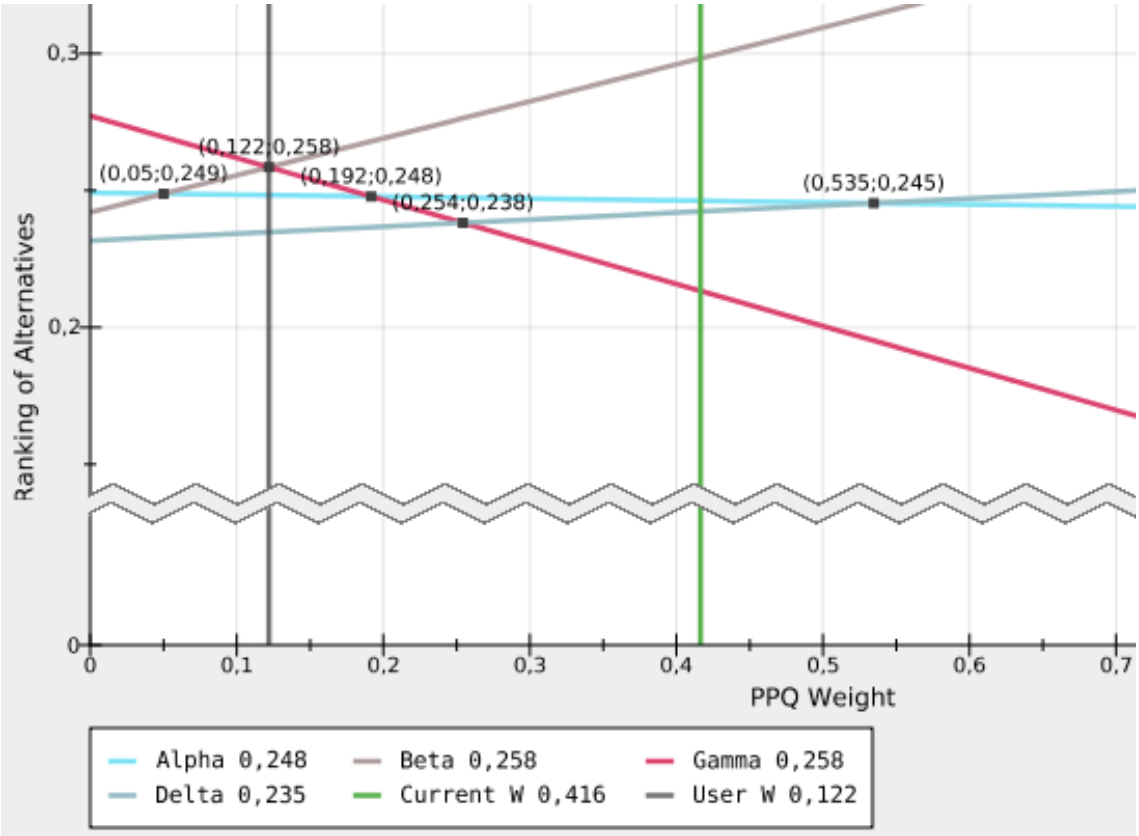


Figure 5.11. Ranking of alternatives using one-at-a-time analysis

Variations in MI and FP do not change the most preferred alternative. When considering the sensitivity on the second level criteria, if under the PPQ criterion the weight of ND increases to 96.5%, CA increases to 93.5% or TT decreases to 2.6% then alpha becomes the top alternative. Under SL only an increase of FL to 59.4% causes gamma to become the best alternative. The alternatives are insensitive to changes in the weights of the sub-criteria of MI. For the sub-criteria of FP, only an increase of PM to 79.7% changes the ranking and gamma becomes the top-ranked alternative.

It is worth noting that although the probabilistic approach shows that each alternative can be the most preferred, using the one-at-a-time analysis delta is never the best alternative.

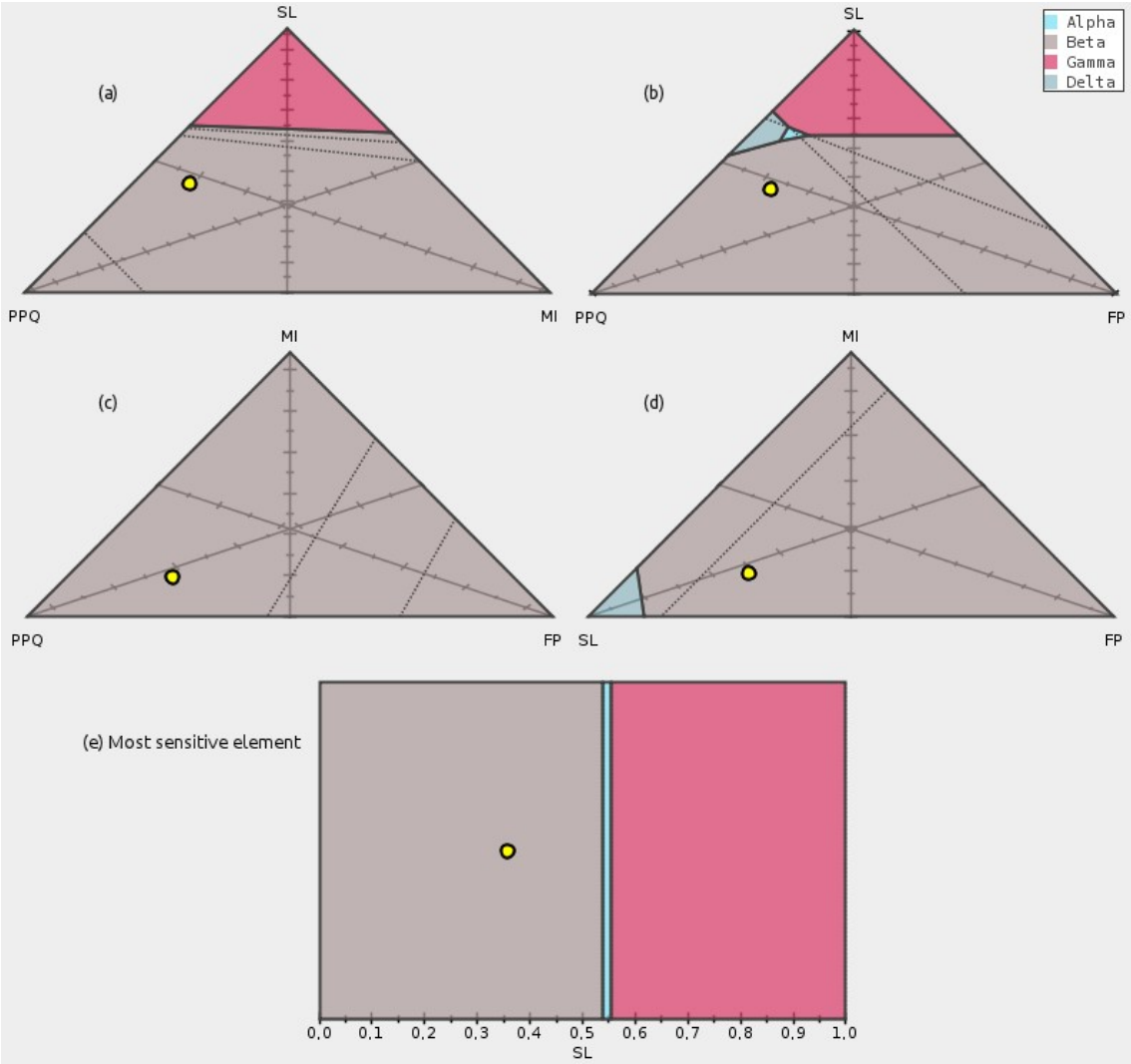


Figure 5.12. Ranking of alternatives using the mathematical SA

Three simultaneous changes were considered when using the mathematical analysis. Figure 5.12 shows the charts generated when the first level criteria is analysed and the most sensitive element identified by PriEsT. The sensitivity coefficients for each chart are presented in Table 5.3.

The lowest OPSC is present in chart (d), however the rank reversal probability is just 2%. The most sensitive element, represented by chart (e), corresponds to the SL criterion with OPSC and RR% of 0.180 and 46.3% respectively. In other words, the smallest change to produce a rank reversal and cause delta to be the most preferred alternative is found in (d), although (e) represents a higher probability of rank reversal when the criteria weights are perturbed.

Chart	*OPSC*	*RR%*	*SC*
(a)	0.257	14.7%	1.172
(b)	0.184	16.9%	1.203
(c)	0.544	0%	1
(d)	0.174	2%	1.102
(e)	0.180	46.3%	1.861

Table 5.3. Sensitivity coefficients

Chart (b) represents all the combinations of weights of PPQ, SL and FP while the weight of MI is kept constant at its original value of 9.5%, hence, the sum of PPQ, SL and FP is fixed at 90.5%. When FP and PPQ are decreased and MI is increased, gamma, alpha and delta may become the most preferred alternatives. By interacting with the chart, the combination of weights that cause a rank reversal can be identified. Table 5.4 shows an example of this situation.

PPQ	*SL*	*MI*	*FP*	*#1*	*#2*	*#3*	*#4*
0.416	0.358	0.095	0.131	beta 0.298	alpha 0.246	delta 0.242	gamma 0.213
0.281	0.550	0.095	0.073	alpha 0.252	delta 0.251	gamma 0.250	beta 0.246
0.308	0.555	0.095	0.041	delta 0.255	alpha 0.253	gamma 0.249	beta 0.243
0.245	0.550	0.095	0.110	gamma 0.253	alpha 0.251	beta 0.249	delta 0.246

Table 5.4. Ranking of suppliers with different weight combinations

Performing the same analysis for the second-level criteria, the most significant result is when PU, FL and PF (sub-criteria of SL) are analysed. For this triple, OPSC is 0.477 and RR% is 16.6% and gamma may become the first alternative if PU and PF are decreased and FL is increased.

5.6 Summary

This chapter has discussed the design and implementation of three different methods for performing SA. In addition, a brief discussion on the advantages

and limitations of each method was presented. The one-at-a-time method provides a fast and easy to use approach for analysing a decision problem, however, only one perturbation can be analysed. The simulation technique allows the decision maker to investigate the ranking of alternatives when the weights of any number of decision elements are modified. This approach has proven to be useful to analyse a problem exploring all possible combinations of weights in all levels of the hierarchy. The mathematical algorithm can analyse any number of decision elements in the same level of the hierarchy and also presents three sensitivity coefficients (OPSC, RR% and SC), however, only up to three elements can be visualised in chart format. The projection of the three-dimensional allowable region into a two-dimensional region provides the user with an easy tool to interpret the results.

In addition, the problem of selecting a single solution from a set of non-dominated solutions generated from the input judgements has been achieved via a novel method that uses the mathematical SA algorithm and sensitivity coefficients.

The chapter ends with the presentation of a SSP (supplier selection problem) case study. The use of the three SA methods allows investigation of each possible scenario where the rank of the best alternative may be reversed. The analysis has shown the usefulness of inspecting the problem with different SA methods. The probabilistic approach showed that each alternative may be the most preferred, although two of them were more likely to be selected. Further analysis was carried out with the one-at-a-time and mathematical algorithms to determine under what conditions the ranking of alternatives may be modified.

Chapter 6
Platform Portability

This chapter discusses the process of porting PriEsT to web and mobile platforms. The web platform was covered by developing a Java applet whilst for the mobile platform two versions were created, one for J2ME-enabled devices and one for the Android operating system. The chapter ends by discussing briefly the challenges faced during the implementation process. Specific tests and evaluation of these versions are presented in Chapter 7.

6.1 Portability Context

In recent years, there has been an increasing interest in web applications with examples such as Google family of web applications, Microsoft Office 365, social networks, media, and many others. Similarly, mobile phones are increasingly being equipped with more processing power, memory and functionality. At the same time, they are becoming the main computing device for many users. Individuals carry them most of the time, and in areas of field activity such as sales and technical support, smart phones are gradually replacing the use of laptops.

There appears a natural evolution of tool provision on web and mobile platforms; however this is not yet apparent in MCDM and AHP software. Whilst several tools support AHP and PC as discussed in Section 3.2, only Web-HIPRE (Mustajoki, 2000), MakeItRational (MakeItRational, 2012) and V.I.S.A. (SIMUL8, 2012) are web-based (other tools only support Windows); there appears to exist no mobile version of AHP software other than the version developed here.

The re-engineering process carried out in the PriEsT tool and the architecture design choices detailed in Chapter 4 enables a relatively simple implementation of both a Java applet and a J2ME mobile version by re-using the PriEsT library and implementing a user interface for the mobile platform, as Java applets can reuse the Swing user interface of a desktop application. In addition, as the

PriEsT desktop application was implemented using standard Java libraries, it is platform independent as it only needs the JVM to run on any operating system. The dependency of the different versions of PriEsT is represented in the diagram of Figure 6.1.

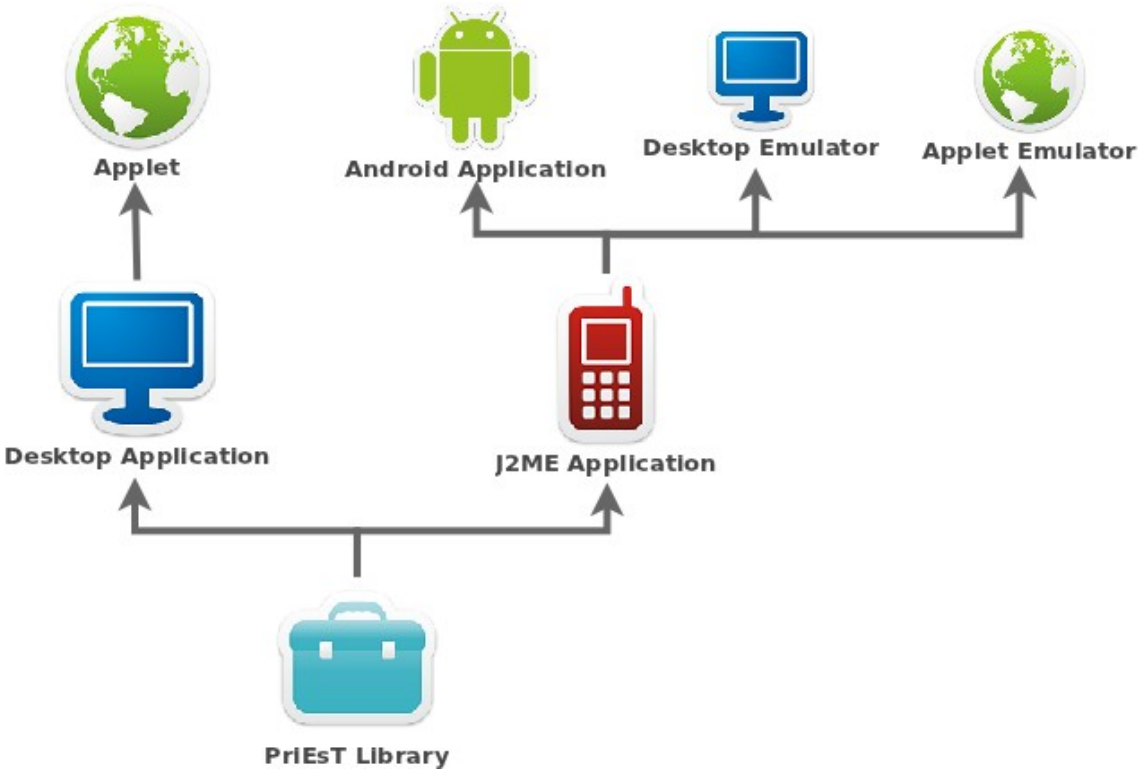


Figure 6.1. Dependency of PriEsT Implementations

6.2 Web Provision

In order to deliver a web version of PriEsT a Java applet was developed. As the PriEsT library and the PriEsT desktop application were implemented using the standard Java Swing framework, all source code was reused to create the applet version. As a consequence, the resulting applet includes all features present in the desktop version.

To implement the applet, only one additional Java class was developed and a web page was created to host the applet. This shows the benefits that may be derived from a design with high cohesion, low coupling and clearly separated

concerns.

In order to test the applet, it was published in the personal pages website available in the School of Computer Science network[7]. The same set of tests used to evaluate the desktop application were executed to evaluate the applet, these tests are included in Appendix B.

6.3 Mobile Provision

6.3.1 J2ME Port

A mobile version of PriEsT was implemented for the J2ME platform allowing PriEsT to be run on any mobile device that supports MIDP.

The design and creation of the user-interface was done using the LWUIT toolkit. This approach has the advantage of decoupling the user-interface from the source code, that is, modifications to the GUI may be made without changing the code of the application.

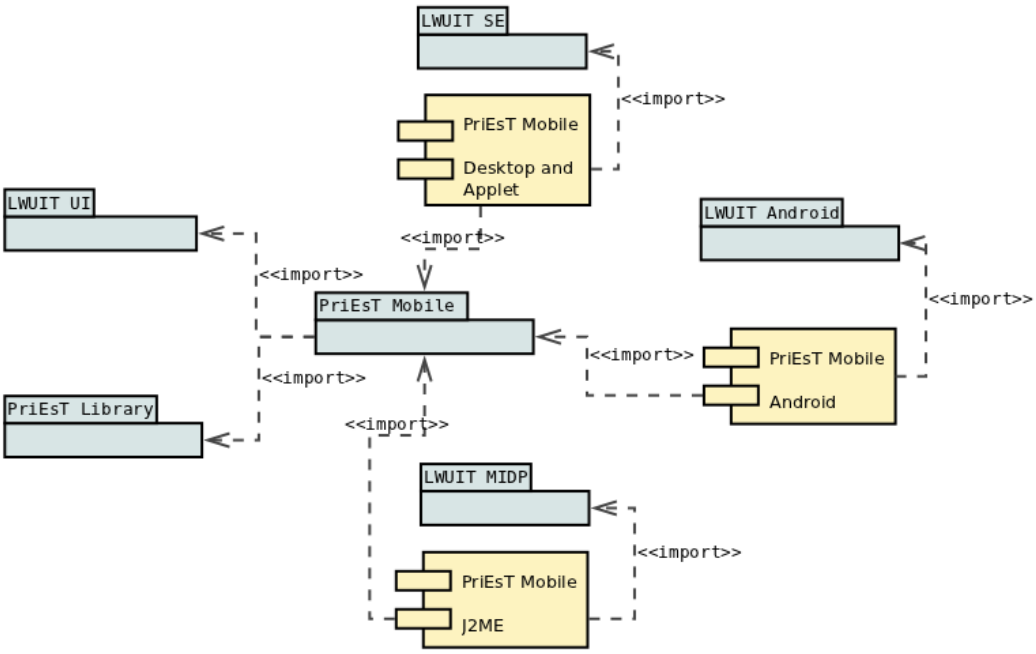


Figure 6.2. Package Diagram for PriEsT Mobile

7 Currently the applet is available online at: http://www2.cs.man.ac.uk/~bertuzzr/

Additionally, by using LWUIT the implementation process can be focused on the application logic rather than on GUI components.

In addition, LWUIT takes care of the implementation-specific issues on each mobile platform such as screen size, memory, APIs, hardware keys, touch screens, etc. resulting in an application that will have the same look and feel on any device, as can be seen on Figure 6.3.

The architecture of the mobile application is similar to the architecture of the desktop application, using the MVC pattern. The PriEsT library was used as the model of the architecture; the GUI was designed end implemented independently using the LWUIT toolkit; and a controller class was implemented to manage the interaction between the GUI and the PriEsT library.

(a) PriEsT Mobile running in the Desktop Emulator

(b) PriEsT Mobile running in a Nokia 5800 smart phone

Figure 6.3. PriEsT Mobile

In addition to the J2ME mobile version, two other applications were also developed on top of the mobile version. LWUIT also supports Java SE, hence the controller and GUI were reused to implement both a desktop and an applet

version of the mobile application. In this way, the mobile version can be tested and tried with no need for a real device. The architecture design is shown in a simplified form in Figure 6.2 while the layout of this dependency is represented in Figure 6.1.

6.3.2 Limitations

A set of features present in PriEsT were removed from the mobile application because of screen size constraints and usability. These features are the graph view, equaliser view and visualising multiple results from methods.

The graph view presents the PC matrix as a graph allowing the user to inspect the intransitive judgements and cycles, but it requires significant space in the screen to draw all nodes, lines and weights, hence, it was removed from the mobile version.

The equaliser view presents the PC matrix as a list of pairwise comparisons allowing the decision maker to inspect the the inconsistency between direct and indirect judgements. For the same reason as above, this feature has been left out of the mobile version.

The last missing feature is the list of all results elicited from a method, especially the optimisation methods. When a method produces more than one result, the mobile version automatically picks the first result and ignores the rest. In order to show all results, which may be as many as hundreds, a large list would be required but in a mobile device such a big list may be cumbersome to use and clutter the user interface.

6.3.3 Android Port

PriEsT Mobile was also ported to the Android platform. Although Android does not support J2ME, it does support Java and, fortunately, LWUIT also supports Android. In order to create the Android port, PriEsT was compiled using the Android SDK and tested using an emulator and a real device (Samsung Galaxy SII). No modifications were necessary, as the architecture of PriEsT allowed a straightforward compilation and creation of an installer package.

6.4 Challenges faced in the implementation

The PriEsT library uses two external libraries, Jama and JMetal. Jama is used to work with matrices while JMetal is used for implementing the optimisation-based elicitation methods. JMetal is incompatible with J2ME as it uses some Java libraries[8] not present in the J2ME platform. For this reason, it was necessary to modify the source code of the library and remove these dependencies.

The source code of the library was analysed and 30 classes were identified as required, directly or indirectly, by PriEsT. Each of these classes was analysed in more detail, external dependencies were removed and then ported to J2ME. To be able to make this port, the tool Retrotranslator was used to back-port the source code from the Java 1.6 version back to the 1.4, which is the version that J2ME supports. In addition, the "java.util" library was also back-ported as it was required both by JMetal and PriEsT. With JMetal and "java.util" ported to J2ME, the optimisation-based methods can be included in PriEsT Mobile.

6.5 Summary

This chapter has discussed the implementation process for porting PriEsT to web and mobile platforms. A brief introduction and motivation were presented and then the web provision was discussed. Next, the architecture for the mobile version of PriEsT was presented and the implementation details of the J2ME and Android version were introduced. The chapter ends describing the challenges and problems faced during this implementation process and they were handled.

8 Some of these libraries include java.util (for using lists, arrays, sorting, etc.) and java.io (for file input/output).

Chapter 7
Project Evaluation

This chapter presents the evaluation of the sensitivity analysis methods and the platform-specific versions of PriEsT. Although testing was conducted throughout the project to ensure the methods and software were correctly implemented (a list of all the tests may be found in Appendix B), a high-level evaluation was performed to obtain indicators about the completeness and performance of the implementation.

7.1 Evaluation of PriEsT

The evaluation of PriEsT was separated into three categories: requirements, performance and portability.

7.1.1 Requirements

The aim of this evaluation section is to produce a traceability matrix that presents the level of fulfilment of the new requirements and also to demonstrate that all the original features of PriEsT were included in the new version after the re-engineering process.

A system testing was performed against the original requirements (Siraj, 2011) and the new requirements covered in Section 4.3 for PriEsT.

Figure B.1 in Appendix B contains a list of the tests performed when PriEsT was first developed by Siraj. These tests were executed against the new versions of PriEsT, desktop and mobile, to certify that all original features are still present in this version.

A brief discussion of the tests that have been ignored, failed or that produced different results as the original plan is given next. Tests that passed in the original plan and passed in the new version are omitted.

- T1-0003. This test is to evaluate group decision making (GDM). The

original version of PriEsT does not support GDM, as a consequence this test have failed in the original test plan. As GDM was not included as a new requirement in the new version, this test was omitted.

- **T1-0006.** This test have passed for the desktop application but was omitted for the mobile application as graph view was not implemented as discussed in Section 6.3.2.
- **T1-0008.** This test evaluates the support for remote input of judgements for GDM. As GDM was not included as a new requirement in the new version, this test was omitted.
- **T1-0009.** Displaying congruence and dissonance measures in the PCM was not implemented in the mobile version as discussed in Section 6.3.2, hence, this test was omitted.
- **T1-0011 and T1-0012.** These tests evaluates the support for highlighting the most inconsistent triple and three-way cycles. For the same reason given above, these tests were omitted for the mobile version of PriEsT.
- **T1-0015 and T1-0016.** These tests have passed for the desktop application and have been omitted for the mobile version. The mobile application automatically selects the first result when an elicitation method produces more than one results for a PCM, as a consequence, the mobile version does not include a chart for plotting results in the TD-NV and TD-TD2 space.
- **T1-0020.** This test, for evaluating support for a web-based application, was failed in the original test plan but have now passed as a web version of PriEsT has been implemented.
- **T1-0023.** Similarly as test T1-0020, this test was failed before but have now passed as a mobile version of PriEsT has been developed.

Table 7.1 shows the results of the system testing performed against the new requirements for PriEsT. All requirements have been fully implemented and tested, although non-functional requirement NR-005 is marked as partial as not

all features of the desktop application are implemented in the mobile version, as discussed in Section 6.3.2.

Test	*Requirements*	*Description*	*Result*
T-001	FR-001	Removal of any decision elements from the model	Passed
T-002	FR-002	Calculate ranking of alternatives automatically	Passed
T-003	FR-003	One-at-a-time SA	Passed
T-004	FR-004	Probabilistic simulation SA	Passed
T-005	FR-005, FR-006	Mathematical SA	Passed
T-006	NR-001	Clear MVC separation	Passed
T-007	NR-002	PriEsT Library	Passed
T-008	NR-003	No native dependencies	Passed
T-009	NR-004	Web application	Passed
T-010	NR-005	Mobile application	Partial
T-011	-	Backwards compatible. Import/export XML files from/to previous version.	Passed

Table 7.1. Traceability matrix for new requirements

The evaluation carried out in Section 3.2.3 was performed again using the new version of PriEsT (desktop and mobile), as presented in Table 7.2. This time, PriEsT obtained an overall score of 31 out of 33 for the desktop application and 29 for the mobile application, in contrast to its previous score of 16.

Feature	*Score* *Desktop / Mobile*	*Description*
A1	3 / 3	PriEsT supports an unlimited number of decision elements and levels in the hierarchy. Elements may be added, edited and removed from the model.
A2	3 / 3	Pairwise comparisons are supported at all levels of the hierarchy.

Feature	*Score* *Desktop / Mobile*	*Description*
A3	2 / 2	Judgements can be entered using only a numerical scale - a verbal scale is not supported.
A4	3 / 1	The consistency index, cardinal and ordinal consistency measures are displayed in both, desktop and mobile applications. However, displaying hints to help the DM to improve overall consistency of the judgements is only supported in the desktop version.
A5	3 / 3	PriEsT supports 11 prioritisation methods which can be selected to derive weights.
A6	3 / 3	Error measures TD, NV and TD2 are presented.
B1	3 / 3	Numerical incremental analysis is supported with an interactive chart. It allows the weights of one element (criterion or alternative) to be changed.
B2	3 / 3	Probabilistic simulations are supported. Uniform, gamma and triangular probability distributions are included.
B3	3 / 3	The allowable range where rank reversals occur is included in the one-at-a-time and mathematical SA.
B4	2 / 2	Sensitivity coefficients are provided, however, the most sensitive elements are not identified automatically. The user has to iterate through all nodes in the decision problem to find the most sensitive decision element.
B5	3 / 3	Any number of elements can be analysed using the probabilistic simulation or mathematical SA. However, a chart is not presented when more than three elements are analysed using the mathematical SA.

Table 7.2. PriEsT v2 Software Evaluation

7.1.2 Performance

Performance tests were carried out to investigate how the re-engineered version of PriEsT (PriEsT v2) compares to the previous one (PriEsT v1). Two items were measured, the start-up time and the time required to elicit all nodes in a decision problem.

To measure the start-up time, both applications were executed three times before the actual test in order for the operating system to load the libraries need by the application, specially in the case of QT libraries, and in this way measure only the time needed by the application to launch.

The test was performed three times on two different operating systems and hardware and the average time was calculated. Figure 7.1 shows the results from this test. In both cases, PriEsT v2 started in around half the time compared to PriEsT v1.

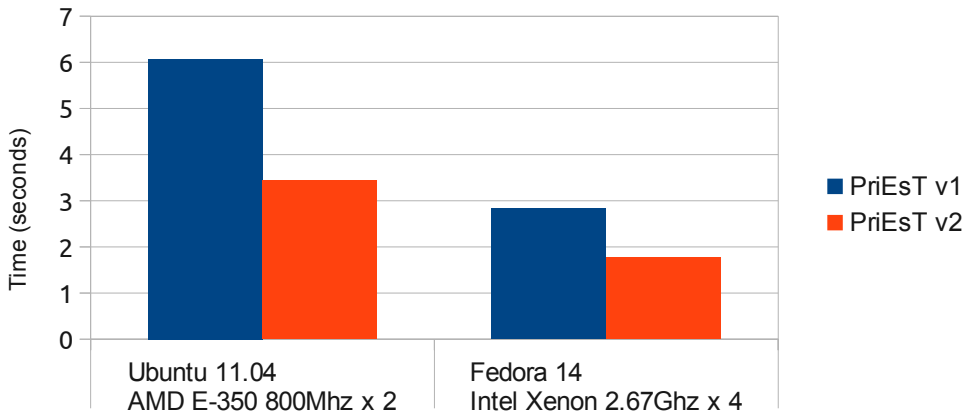


Figure 7.1. Start-up time

To measure the time to solve a decision problem, the application was started and the decision problem discussed in Section 5.5 was imported from an XML file. This problem contains 12 criteria distributed into two levels and four alternatives. All nodes were elicited once using the “Rebuild All” button before the actual test was performed. The problem was solved three times using all elicitation methods, the execution time was recorded and then the average time was calculated.

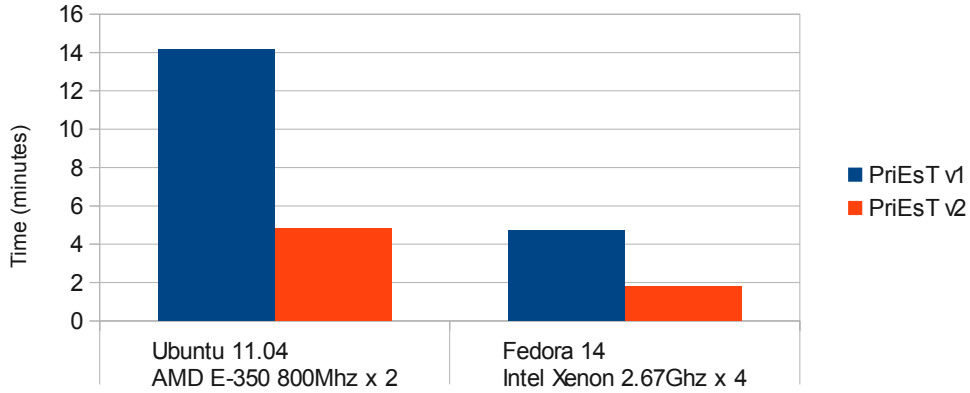


Figure 7.2. Performance comparison

For PriEsT v1, only the time to elicit the nodes was considered and not the time to rank the options, as it is implemented as a separated operation. The time registered for PriEsT v2 includes the elicitation and the ranking process.

The results of this test are shown in Figure 7.2. In a dual core CPU of 800Mhz, PriEsT v2 takes 5 minutes while PriEsT v1 takes 14 minutes to solve the same problem; in a CPU with 4 cores of 2.67Ghz, v2 takes 2 minutes while v1 takes 5 minutes to solve the problem. On both of these tests, v2 proves to be approximately twice as fast as v1.

A test to evaluate the performance of the probabilistic simulation in a mobile device was carried out. The probabilistic simulation was performed with the default value of 10000 iterations, uniform distribution and all 12 criteria were included in the analysis. The test was carried out in a smart-phone Nokia 5800 XpressMusic. This touch-screen device has a processor ARM 11 434 MHz with operating system Symbian OS 9.5. The experiment showed an average time to complete the simulation of 10.4 seconds, which would seem to be acceptable for a mobile device processor.

Additionally, a performance test was executed in the mobile device to evaluate the response time of the elicitation methods versus the PC matrix size.

Figure 7.3 shows the results of the elicitation test using a logarithmic scale. GM, EV and NCS have a similar performance and produce results in less than 20 milliseconds independently of the matrix size.

For matrix size of 7 or above,the performance of EAST is greatly degraded, taking around 400 seconds to solve a matrix with 7 elements. For 8 and 9 elements, this test was skipped as it would require a significant time to complete.

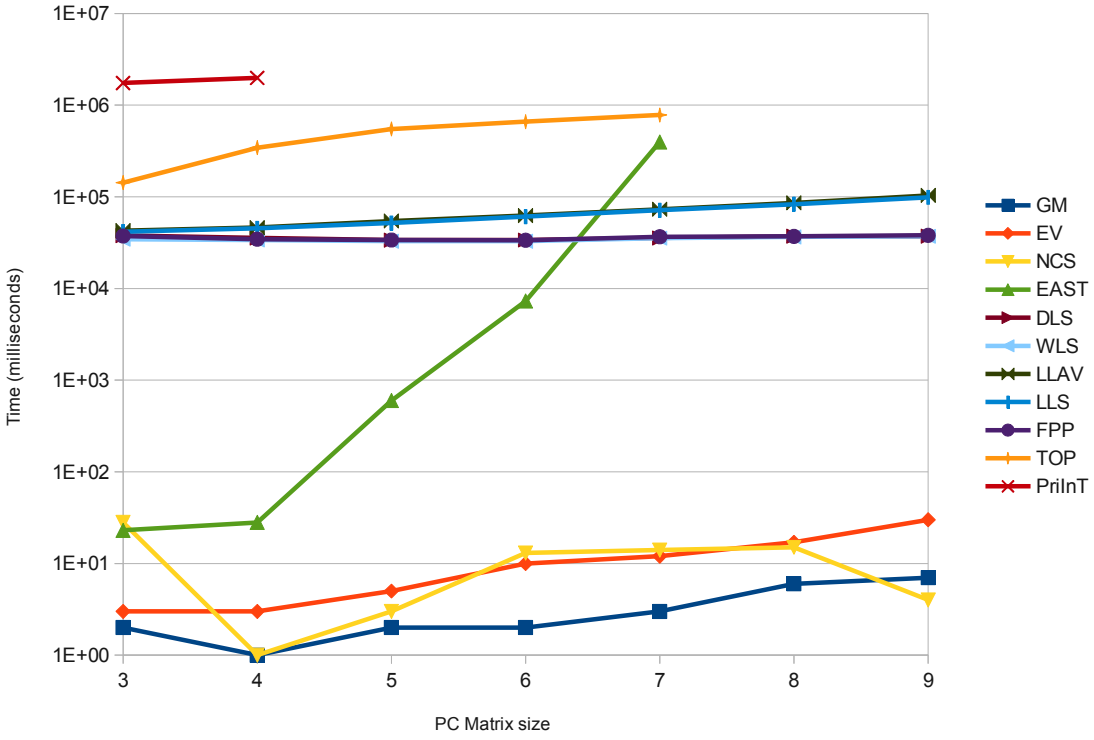


Figure 7.3. Performance of elicitation methods for PriEsT Mobile

Optimisation-based methods DLS, WLS, LLAV and FPP present similar performance, taking approximately between 30 and 100 seconds to solve a matrix of 3 and 9 elements, respectively. TOP and PrInT methods that take a considerable time in comparison with other optimisation methods, making them unsuitable for mobile devices with slow processors.

7.1.3 Cross-Platform Portability

PriEsT, in its desktop and applet version, was executed on the three major operating systems Windows, Mac and Linux (NetMarketShare, 2012). Windows XP was selected from the Windows family of operating systems as it is the most

used version as of 2012 (NetMarketShare, 2012); for Max and Linux, Mac OS X v10 and Ubuntu Linux 11.04 were selected.

A decision problem aimed at evaluating emerging technologies in the semiconductor industry (Chen, 2009) was selected to test PriEsT on the three operating systems. The problem was loaded from an XML file, solved and then analysed using SA.

In the three platforms, the results were the same and all features worked properly. Appendix D contains screen-shots of PriEsT running on each operating system.

7.2 Evaluation of Sensitivity Analysis

In order to evaluate the SA methods, two analyses were carried out: a performance test and a comparison evaluation to investigate the accuracy of the algorithms in identifying rank reversals.

To perform these tests, two Monte-Carlo simulations were executed. In the first simulation, for each iteration a decision problem was created with *n* criteria, ranging from 3 to 12, and 9 alternatives[9]. In the second simulation, the number of criteria was fixed at 9, and the alternatives ranged from 3 to 12. In both simulations, 3000 problems where generated for each iteration; all judgements were generated randomly using the scale 1-9.

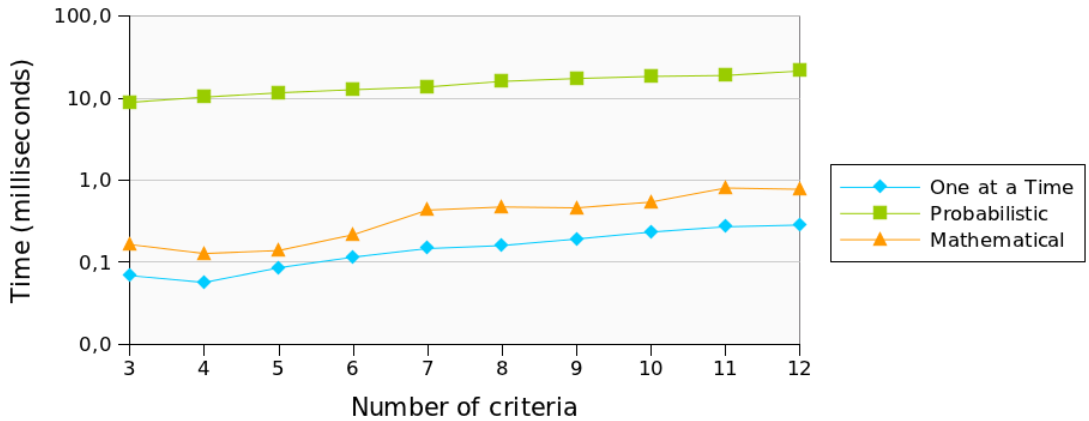
For each iteration, the problem was solved using the GM method and then an SA (using all three methods) was conducted on the criteria weights. For the probabilistic simulation algorithm, the default 10000 uniform random simulations were used. The execution time and results from each SA algorithm were recorded and are analysed next.

7.2.1 Performance

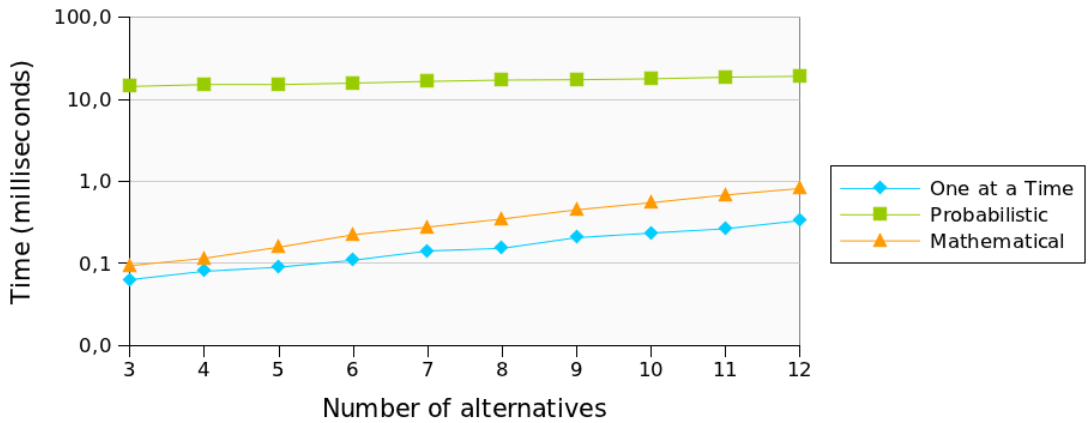
The average time was calculated from the results of the Monte-Carlo

9 Saaty (Saaty, 1977) defined 9 as the limit for any PC matrix based on a cognitive limitation of the mind which cannot handle more than 7±2 simultaneous comparisons.

simulations to compare the execution time of each SA method. The test was performed on a machine with an Intel Core 2 Duo CPU running at 3.00GHz.



(a) Computation time vs criteria



(b) Computation time vs alternatives

Figure 7.4. Computation time for SA methods

Figure 7.4a shows the results of the test with 9 alternatives and the number of criteria ranging from 3 to 12, while Figure 7.4b gives results for the simulation when the number of criteria is fixed at 9 and alternatives range from 3 to 12.

One-at-a-time and mathematical methods take the lowest amount of computational time, below 3 milliseconds, and appear to be insensitive to the number of alternatives or criteria.

The probabilistic approach takes more processing time than the other two methods and shows a linear relation between the number of alternatives or

criteria and the computation time. However, the time required to perform SA using this method is still relatively low, as approximately 22 milliseconds were required to perform a probabilistic simulation with 12 criteria and 9 alternatives, as shown in Figure 7.4a.

The impressively low time for the probabilistic simulation algorithm, which performed 10000 simulations, may be explained by the fact that a warm-up session was executed before the actual test. This session consisted in running one Monte-Carlo iteration to allow the JVM to load all the classes necessary to prevent the time used by the class-loader to affect the results. This session may have triggered the JIT compiler to compile the code of the probabilistic algorithm into native code causing an increase in the performance.

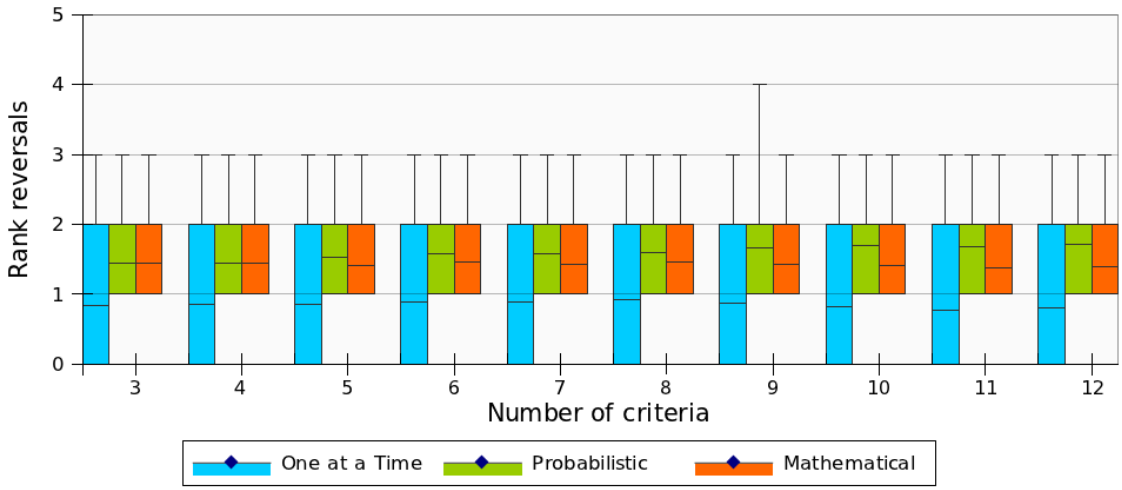
7.2.2 Methods Comparison

This analysis was carried out to compare how the three SA algorithms behave when finding rank reversal conditions.

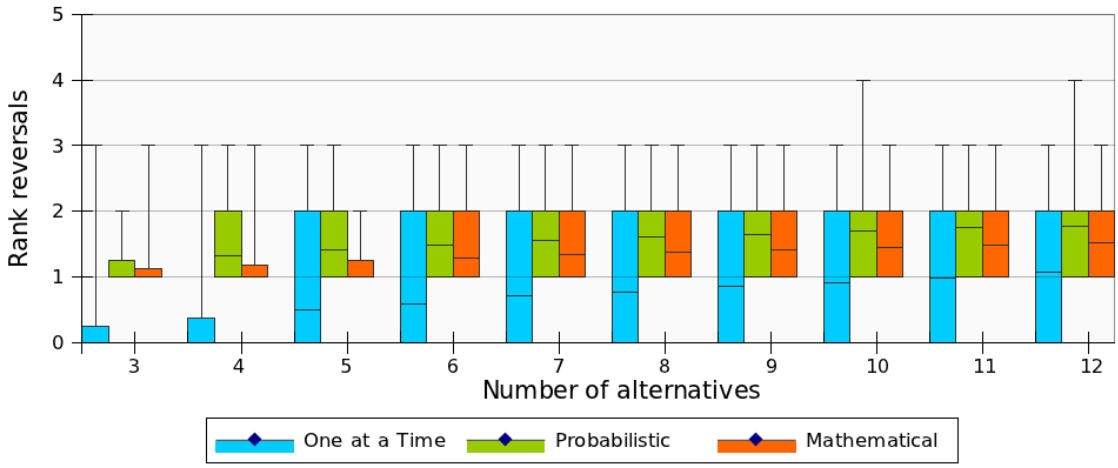
For each iteration of the Monte-Carlo simulations the ranking of alternatives was calculated and the three SA algorithms were executed. As the output of the algorithms cannot be compared directly as they produce different kind of results, the analysis focused on finding how many alternatives may be the most preferred according to each algorithm.

Under this analysis, the most effective algorithm is the one that finds the highest number of rank reversals that causes the top-ranked alternative to change. Rank reversals between other alternatives are ignored.

Figure 7.5 gives the results of the simulation. The y-axis represents the number of rank reversals while the x-axis represent the number of criteria (Figure 7.5a) and number of alternatives (Figure 7.5b). As the simulation was repeated 3000 times for each value of the x-axis, the indicators minimum, maximum, quartile 1, quartile 3 and mean were calculated.



(a) Rank reversals vs criteria



(b) Rank reversals vs alternatives

Figure 7.5. SA Method Comparison

Each series represents 5 indicators: minimum, maximum, quartile 1, quartile 3 and mean number of rank reversals.

The chart (a) shows that probabilistic and mathematical SA perform very similarly when the number of alternatives is constant. However, as the number of criteria increases, the mathematical algorithm presents a slightly lower average of rank reversals (represented by the horizontal line inside each box). In addition, for n=9, the maximum number of rank reversals reported by the probabilistic approach is 4, while the mathematical algorithm reported 3.

The one-at-a-time analysis is outperformed by the other approaches, on average it finds 0.8 rank reversals whilst the other methods find 1.5. Furthermore, the minimum number of reversals reported by this algorithm is 0,

while the results for the probabilistic and mathematical methods are always 1.

A similar interpretation can be obtained from the chart in Figure 7.5b, although for 3 alternatives the probabilistic approach reported a maximum of 2 rank reversals, while both one-at-a-time and mathematical algorithms reported 3. The one-at-a-time algorithm reports a minimum of 0 and a lower average of rank reversals than the other methods. For n=10 and n=12, the probabilistic simulation method reported a maximum of 4 rank reversals, while other methods reported 3.

From both charts we can conclude that the probabilistic and mathematical methods have similar performance in finding rank reversals. As more alternatives or criteria are added to the problem, the average number of rank reversals is slightly higher for the probabilistic approach. This difference may be explained because the probabilistic simulation algorithm analyses all decision elements in all levels of the hierarchy at the same time, while the mathematical approach analyses elements that are located on the same level.

The one-at-a-time SA method, as it only analyses one element at a time, has a poorer performance at finding rank reversals. Although it is the easiest to use, we recommend its usage in small problems or when the decision maker is uncertain about the weights of one or two elements, as it may overlook conditions that cause the ranking of the top alternative to change.

7.3 Summary

This chapter has covered the analysis performed in order to evaluate the software and algorithms developed in this project.

An evaluation of the requirements, performance and portability of the re-engineered version of PriEsT was carried out.

System testing was done to certify that all features present in the original version of PriEsT are present in the new version. Additionally, the tool has been tested successfully in different operating systems.

Performance tests have shown that the re-engineering process, in addition to

improving the cohesion of the source code and architecture, it has improved the response time of the application (by approximately a factor of 2 on limited tests). However, testing has also shown that some optimisation-based elicitation methods, such as TOP and PrInT, require too much computational power to be suitable for mobile devices.

Experimental results have shown acceptable performance for the three SA methods. One-at-a-time and mathematical methods are insensitive to problem size, while the probabilistic simulation method showed a linear relation between computation time and problem size.

The probabilistic simulation and mathematical methods have reported a similar effectiveness in finding rank reversals and outperform the one-at-a-time algorithm.

Chapter 8
Conclusion and Future Work

This chapter presents a summary of the contributions of this research, reflections about the project progress and proposes areas for further investigation.

8.1 Summary

Sensitivity analysis (SA) is an important stage in multi-criteria decision making (MCDM). A research has been carried out and three main types of SA has been identified: one-at-a-time, probabilistic simulation and mathematical modelling. One-at-a-time sensitivity analysis, also named as numerical incremental analysis, is the most commonly used method in research papers and decision support tools.

Several software tools were analysed and compared in terms of AHP and SA features. Although it does not support SA, the decision support tool PriEsT obtained the highest score and was selected for further development. The main goal of the development was to implement the three types of SA.

After and in-depth analysis of PriEsT, the tool was re-engineered to make further development easier. A new design and architecture were introduced. All the application logic was separated from the user interface and the PriEsT library was developed. Next, a new GUI, with similar look and feel, was implemented using only Java libraries. Thanks to the re-engineering process, a clear model-view separation was achieved, and the cohesion and coupling of the application was improved.

The next step was to design and implement the SA algorithms. Each of them was designed and implemented in PriEsT followed by a discussion about the advantages and limitations. In addition, two sensitivity coefficients were implemented for the mathematical modelling algorithm and a method for finding the most sensitive element was presented. Furthermore, using the

mathematical SA method, an approach is developed for selecting a single solution when elicitation methods, such as PrInT or TOP, generate more than one.

In order to show the benefits of each SA method, a case study for selecting and evaluating suppliers was presented. The probabilistic method is useful in getting a broad view of the problem and identifying which alternatives are more likely to be selected, while the one-at-a-time and mathematical algorithms can identify accurately what combinations of weights causes the best alternative to change.

Once the SA methods were finished, PriEsT was ported to web and mobile platforms. The clear MVC separation and the usage of standard Java libraries only was very advantageous when porting the application, as a Java applet was built by fully reusing the desktop application. A mobile version was implemented by reusing the PriEsT library and creating a new GUI.

Finally, the project was evaluated. All platform-specific versions of the application were tested for requirements and performance. In addition, performance tests were conducted to evaluate the three SA algorithms. These tests showed that the computational power required by the algorithms is almost insensitive to the number of decision elements. In terms of efficacy in detecting rank reversals, the experiments results showed that probabilistic simulation and mathematical modelling have similar performance and both outperform the one-at-a-time method.

8.2 Future Work

The following are areas recommended for further investigation and development.

8.2.1 Enhance One-at-a-time SA Chart

When an SA is conducted using the one-at-a-time approach, if the decision problem contains many alternatives, for instance more than 8, then the chart

becomes cluttered and it is hard to interpret because there are many lines and rank reversal points.

One solution is to allow the user to display the rank reversals for the top alternative only. In this way, the amount of information in the chart is minimised.

Another solution is to allow the user to select which alternatives to present in the chart. This would allow the user to add or remove alternatives from the chart and focus on the alternatives that are important to him/her.

Additionally, enabling zooming in and out may be of value to the user. The chart component supports zooming, but it was disabled to allow the user to click and drag the chart to select different weights. A check-box may be added to enable/disable the zooming feature to allow the user to do both operations.

8.2.2 Include a list with all rank reversals

In order to find all the rank reversal conditions, that is the combination of weights that cause a rank reversal, the user has to analyse each criterion of the decision problem with the one-at-a-time or mathematical SA.

A feature could be added to do this automatically and present the results in a tabular format. Two lists could be added, one for the one-at-a-time and one for the mathematical approach. These lists must present at least the following information: criteria (one criterion or up to three criteria for the one-at-a-time or mathematical method respectively), weights, top alternative. As the mathematical SA supports up to three elements to be analysed, an option should be given to the user to select the number of elements to analyse.

Having such a list could be very helpful for the decision maker to be aware of all the combinations of weights that cause a rank reversal and, in this way, the DM can focus the sensitivity analysis on elements that may affect the final ranking of alternatives.

8.2.3 Calculate sensitivity coefficients for more than three elements

The mathematical SA works in two steps. First, for each alternative, the inequalities defining the allowable range of weights when this alternative is the most preferred are calculated. Second, the inequalities are solved and projected onto a two-dimensional plane.

The sensitivity coefficients are calculated using the two-dimensional plane, as it is easier to compute areas and distances. However, there is one limitation. When more than three elements are analysed a projection into the two-dimensional plane is not possible, hence, the sensitivity coefficients cannot be calculated.

An algorithm should be implemented to calculate the sensitivity coefficients without the need of projecting the polytope into the two-dimensional plane. The polytope is defined by the inequalities produced by the mathematical SA algorithm.

To calculate the OPSC coefficient, the shortest distance from an n-dimensional point (the current weights) to the edges of the polytope must be calculated. Finding the vertices of the polytope may help in this calculation. An algorithm for finding the vertices was proposed by Balinski (Balinski, 1961).

To calculate the TSC coefficient (RR% = 1 – TSC), the hyper-volume of the polytope must be computed. Cohen (Cohen, 1979) developed two algorithms for calculating the volume of an n-dimensional polytope.

8.2.4 Present results for more than three elements

Similarly to the problem discussed in Section 8.2.3, further investigation is needed to find a way to display the results of the mathematical SA in a graphical format when more than three elements are analysed.

8.2.5 Perform SA on the PC Matrix

The three SA algorithms implemented in this project perform analysis on the

weights elicited from the pairwise comparisons (PC). There may be cases where the decision maker wants to know how sensitive the final ranking of alternatives is to changes in the PC judgements. As there are many methods for eliciting weights from a PC matrix, developing an algorithm that works for every method seems unlikely.

The method developed by Chen and Li (Chen, 2011) presented in Section 2.7.3 allows to conduct a SA on the PC matrix using the column-row-orientation elicitation method. As currently PriEsT does not support this elicitation method, an addition to PriEsT is to implement this method and integrate the SA algorithm proposed by Chen and Li.

However, this integration is not straightforward as the method developed by Chen and Li uses the constant-sum pairwise comparisons. Under this approach, judgements a_{ij} are given using a scale from 0 to 100 and the inverse comparison is calculated as $a_{ji} = 100 - a_{ij}$, in contrast to Saaty's method (see Section 2.3) where $a_{ji} = 1 / a_{ij}$, which is the method implemented in PriEsT. As a consequence, the proposed SA algorithm have to be adapted to work with Saaty's method before it can be used in PriEsT.

In addition, the algorithm produces a system of non-linear inequalities defining the feasible range where an alternative is the most preferred, making necessary the use of numeric methods for solving the system, and 3-D plotting libraries for displaying the results when three judgements are analysed as it seems it is not possible to project the three-dimensional object into a two-dimensional plane in the same way as was performed in Section 5.3.2.3.

More research is necessary to identify or develop new algorithms to perform SA on the PC matrix using the elicitation methods included in PriEsT.

In the same way as the one-at-a-time SA was implemented, PriEsT could be extended to allow the user to change the judgements and calculate the new ranking of alternatives.

Similarly, the probabilistic simulation approach can be extended to replace the judgements with random values. For instance, using the method developed by Hauser and Tadikamalla (Hauser, 1996) discussed in Section 2.7.2.

8.2.6 Evaluate the method for selecting a single solution

A novel method was developed and presented in Section 5.4 that addresses the problem of selecting a single solution from a set of non-dominated solutions generated from a PC matrix.

The method works by performing an SA using the mathematical algorithm to obtain the most sensitive decision element for each of the solutions. The solution that produces the most robust final ranking of alternatives is considered to be superior and is selected. Further work is necessary to evaluate if this approach is appropriate for any decision problem or if there are any conditions that prevent its application. For instance, a literature review could identify appropriate case studies for performing this evaluation. In addition, more research is needed to investigate other approaches for solving this problem and compare the outcome of those methods with the method developed in this dissertation.

Additionally, the algorithm for locating the most sensitive element takes into consideration two sensitivity coefficients, OPSC and TSC. The element with the lowest OPSC and TSC is considered as the most sensitive, however, if the lowest OPSC and TSC do not occur on the same element, then these two elements are compared and the one with the lowest $OPSC * TSC$ is considered the most sensitive. More testing and analysis must be carried out to identify if multiplication is a proper solution in each case. For instance, a different approach may be to select the element with the lowest $a * OPSC + b * TSC$, where a and b are constants that have to be discovered by experimentation. Another possible method is to take into consideration the weights of the elements when deciding which is more sensitive.

8.2.7 Implement missing features in the mobile version

Three features of PriEsT were not implemented in the mobile application because of usability issues, as discussed in Section 6.3.2. These features are the graph view, equaliser view and visualising multiple results from methods.

In order to implement these functionalities, some UI designs have to be

developed and evaluated by surveying real users. This is particularly important as the screen size and resolution of a mobile device is considerable smaller than a desktop or laptop computer, hence, bad user-interface design decisions may affect negatively the usability of the mobile application.

8.2.8 Interoperability

PriEsT supports importing and exporting decision problems using custom XML files. For improving interoperability between PriEsT and other decision support tools, XCMDA support should be included. XMCDA (Bigaret, 2012) is a data standard based on XML to represent MCDA problems in a standardised format that can be shared among different applications.

Additionally, to allow the use of PriEsT with spreadsheet applications, an useful feature would be the addition of an export facility for CSV (comma separated values), ODF (open document format) and/or Microsoft Excel file formats.

8.3 Project Plan Fulfilment

The main goal of the project was to enhance a decision support tool with sensitivity analysis, which has been successfully accomplished by adding three different SA algorithms to the PriEsT tool.

In general, a steady progress was maintained throughout the dissertation. Although the earlier phases of the the project were somehow slower than the rest due to the complexity of the methods found during the literature review and the in-depth analysis carried out to understand the architecture and source code of PriEsT, it did not affect the overall planning and progress of the project. The project Gantt chart can be found in Appendix E.

A brief discussion of how each of the objectives of the project was completed in the dissertation follows.

8.3.1 Literature review for Sensitivity Analysis

A literature review was carried out in Chapter 2 to provide the project context

of MCDA and AHP. Section 2.7 discussed existing methods to perform SA. Three families of methods were identified: One-at-a-time Analysis (Section 2.7.1), Probabilistic Simulations (Section 2.7.2) and Mathematical Models (Section 2.7.3).

8.3.2 Decision Support Tools Comparison

Chapter 3 presented a list of desirable features that should be present in decision support tools. An analysis of four software tools was performed in Section 3.2 to evaluate to what extent they comply with such features. The PriEsT tool obtained the highest score and was selected for further development.

8.3.3 Re-engineering PriEsT and SA implementation

The re-engineering process was covered in Chapter 4. An analysis of the current version of PriEsT was carried in Section 4.2; new requirements were introduced in Section 4.3; and Sections 4.4 to 4.7 presented the design and implementation of the new PriEsT tool.

Chapter 5 covered the design and implementation of the SA methods: One-at-a-time (Section 5.1), Probabilistic Simulation (Section 5.2), and Mathematical Modelling (Section 5.3). In addition, a case study was presented in Section 5.5 to show the benefits of these methods.

8.3.4 Platform provision

The platform availability was discussed in Chapter 6. Section 6.2 covered the implementation of a web version of PriEsT,while Section 6.3 covered the implementation of the mobile application.

8.3.5 Project Evaluation

An evaluation of the project was presented in Chapter 7. Section 7.1 presented the evaluation of the PriEsT tool in terms of performance and requirements. Section 7.2 discussed the evaluation of the sensitivity analysis methods

implemented in PriEsT. Performance tests were included in Section 7.2.1 while a comparison of the methods was presented in Section 7.2.2.

8.4 Conclusion

This dissertation has presented research on SA methods for MCDM and their implementation in a decision support tool.

In the background chapter, a discussion of MCDM and sensitivity analysis was presented and three families of SA methods have been identified: one-at-a-time, probabilistic simulation and mathematical modelling.

A software survey was performed to analyse what AHP and SA features are normally included in decision support software tools. None of the software that were considered in the study supports the probabilistic or mathematical SA methods. It was found that software tools usually support the one-at-a-time analysis only.

In order to implement a decision support tool with support for all three SA methods, the PriEsT software was re-engineered. A clear MVC separation was carried out and the PriEsT library was created. Following the re-engineering process, three SA algorithms were designed and implemented. In addition, the tool was ported to web and mobile platforms.

The evaluation of the SA algorithms has shown that the probabilistic simulation and mathematical approaches outperform the one-at-a-time method, as the first two are able to identify more rank reversal conditions than the latter.

Considering that the one-at-a-time analysis is the most popular SA method in research papers and decision support tools, this situation raises a concern about the limited tools available to decision makers when conducting sensitivity analysis on their decisions. We believe that PriEsT may be of real help in this matter, as the three methods may be used together to provide a more robust SA.

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Appendix A
UML Design

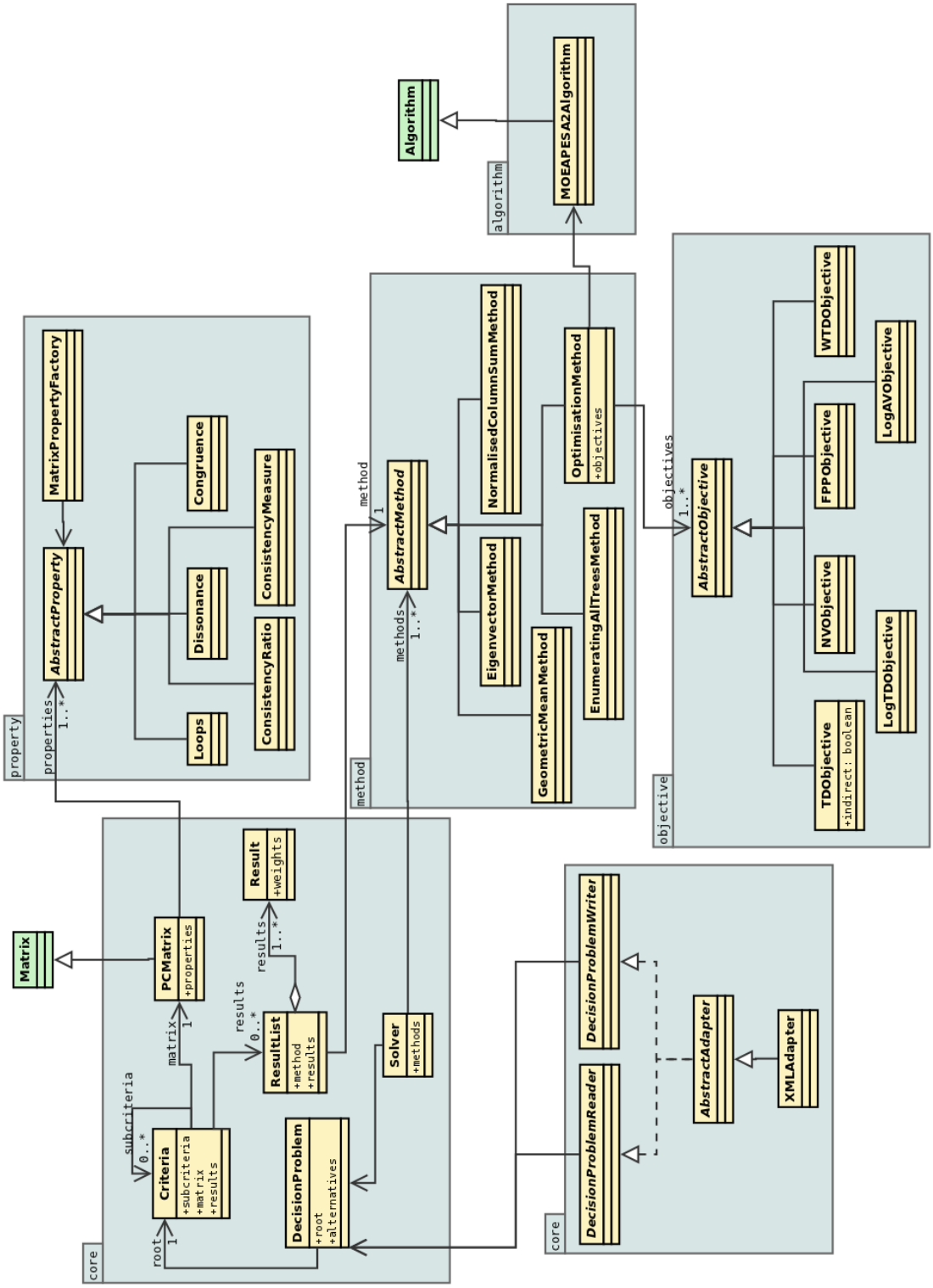

Figure A.1. Class Diagram for PriEsT Library

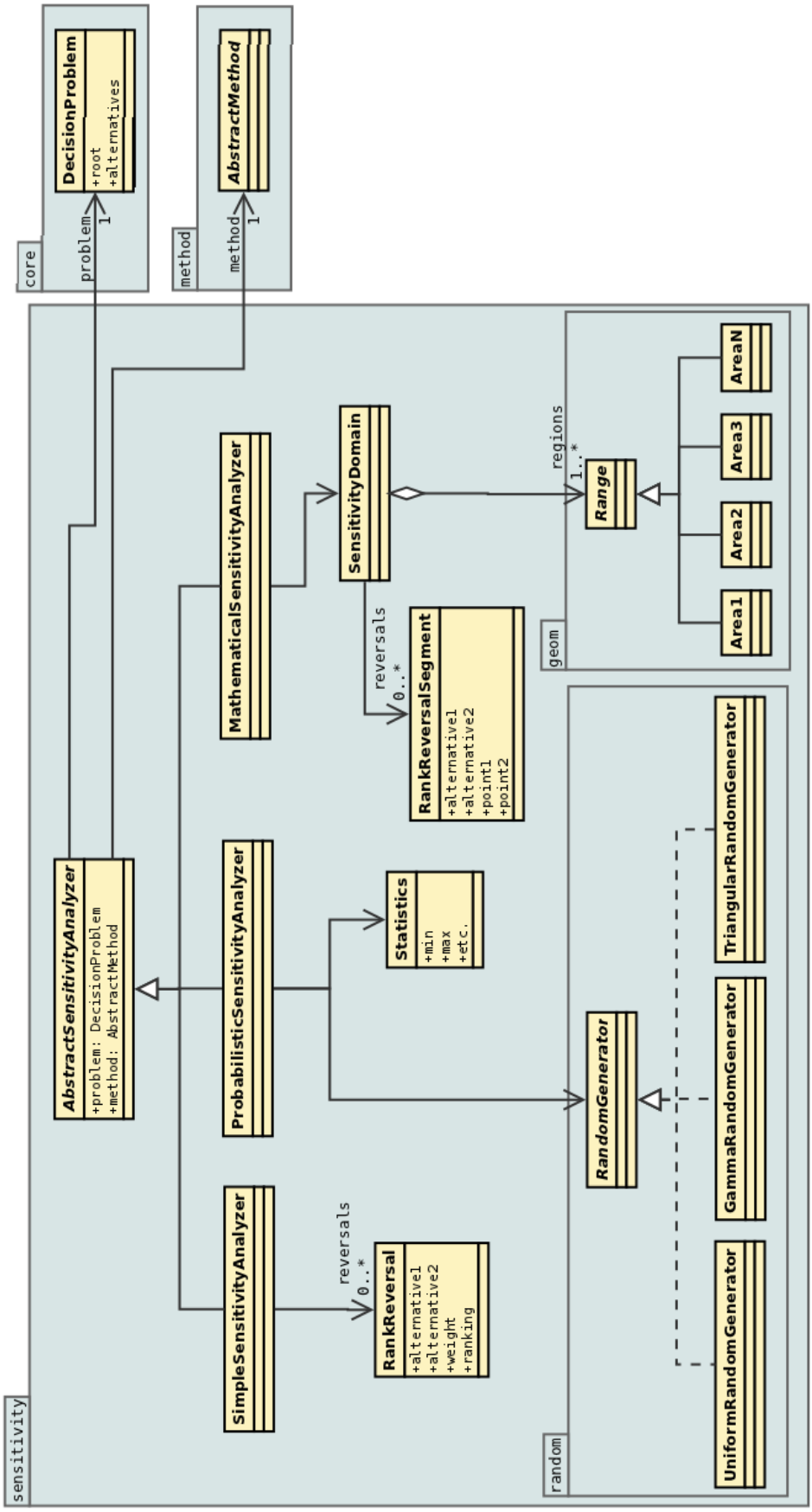

Figure A.2. Class Diagram for Sensitivity Analysis

Appendix B
Unit Tests

Test No.	*Requirements Covered*	*Test Details*	*Result*
T1-0001	FR-01-001 FR-01-004	Editable criteria and hierarchical view	ok
T1-0002	FR-01-002	Editable alternatives	ok
T1-0003	FR-01-003	Editable agents for group decisions	not supported
T1-0004	FR-01-005	Preference equivalence enable/disable	ok
T1-0005	FR-02-001	Table view for entry	ok
T1-0006	FR-02-002	Graph view for entry	read-only
T1-0007	FR-02-003	Entering judgments on measurement scale	read-only
T1-0008	FR-02-004	Entering judgments remotely using Web service	not supported
T1-0009	FR-03-001, FR-03-002	Show congruence & dissonance matrices	ok
T1-0010	FR-03-005	Display CR, CM, L, Θ and Ψ	ok
T1-0011	FR-03-003	Highlighting the most inconsistent triple	ok
T1-0012	FR-03-004	Highlighting all three-way cycles	ok
T1-0013	FR-04-001	Selecting different prioritization methods	ok
T1-0014	FR-04-002	Visualizing priority vectors	ok
T1-0015	FR-04-003	Plotting results in the TD vs. NV space	ok
T1-0016	FR-04-004	Plotting results in the TD vs. TD2 space	ok
T1-0017	FR-04-005	Finding overall ranking using Additive AHP	ok
T1-0018	FR-05-001	Exporting data to an XML file	ok
T1-0019	FR-05-001	Importing saved data from XML	ok
T1-0020	FR-05-002	Web-based application	not supported
T1-0021	NR-01-001	Cross-platform working Run on both Windows XP and Linux	ok
T1-0022	NR-01-002	Run PriEsT on Intel-based PC with Core2Duo T5500	acceptable
T1-0023	NR-01-003	Front-end: Web and Phone Applications	not supported

Figure B.1. System testing performed on previous version of PriEsT (Siraj, 2011)

100,00 %

All 80 tests passed.(7,145 s)

- com.man.priest.core.CriterionTest passed
  - testGetName passed (0,027 s)
  - testSetName passed (0,001 s)
  - testGetParent passed (0,001 s)
  - testSetParent passed (0,002 s)
  - testAddCriterion_Criterion passed (0,001 s)
  - testAddCriterion_String passed (0,0 s)
  - testGetChildren passed (0,001 s)
  - testGetCriterion passed (0,0 s)
  - testGetChildrenCount passed (0,0 s)
  - testHasChildren passed (0,001 s)
  - testRemoveAllCriteria passed (0,001 s)
  - testIndexOf passed (0,001 s)
  - testSetPCMatrix passed (0,025 s)
  - testGetPCMatrix passed (0,0 s)
  - testGetResult passed (0,002 s)
  - testGetResults passed (0,01 s)
  - testClearResults passed (0,001 s)
- com.man.priest.core.DecisionProblemTest passed
  - testGetGoal passed (0,019 s)
  - testSetGoal passed (0,001 s)
  - testAddAlternative passed (0,001 s)
  - testSetAlternatives passed (0,002 s)
  - testGetAlternatives passed (0,001 s)
  - testGetAlternativeCount passed (0,0 s)
  - testGetAlternative passed (0,0 s)
  - testRemoveAlternative_int passed (0,0 s)
  - testRemoveAllAlternatives passed (0,001 s)
  - testGetRootNode passed (0,001 s)
  - testGetRanking passed (0,186 s)
  - testGetTD passed (0,002 s)
  - testGetTD2 passed (0,002 s)
  - testGetNV passed (0,002 s)
  - testGetNodeCount passed (0,002 s)
- com.man.priest.core.PCMatrixTest passed
  - testGetProperties passed (0,035 s)
  - testGetPropertySize passed (0,001 s)
  - testGetProperty passed (0,007 s)
  - testSet passed (0,001 s)
  - testIsInconsistent passed (0,003 s)
  - testGetInconsistentCycle passed (0,001 s)
  - testGetCycleSize passed (0,001 s)
  - testGetCongruence passed (0,001 s)
  - testGetMaxCongruence passed (0,001 s)
  - testGetDissonance passed (0,0 s)

Figure B.2. Unit testing performed on PriEsT

- com.man.priest.core.SolverTest passed
  - testAddMethod passed (0,016 s)
  - testRemoveMethod passed (0,001 s)
  - testGetMethodCount passed (0,002 s)
  - testSolve passed (0,083 s)
- com.man.priest.io.XMLAdapterTest passed
  - testAccept passed (0,015 s)
  - testWrite passed (0,155 s)
  - testRead passed (0,073 s)
- com.man.priest.sensitivity.MathematicalSATest passed
  - testAnalyze passed (0,061 s)
- com.man.priest.sensitivity.ProbabilisticSATest passed
  - testSetRandomGenerator passed (0,012 s)
  - testGetRandomGenerator passed (0,0 s)
  - testGetIterations passed (0,001 s)
  - testSetIterations passed (0,0 s)
  - testSetPreserveRankOrder passed (0,0 s)
  - testSimulate passed (0,94 s)
- com.man.priest.sensitivity.SensitivityDomainTest passed
  - testSetWeightsSum passed (0,04 s)
  - testGetWeightsSum passed (0,001 s)
  - testGetCriterion passed (0,0 s)
  - testGetDimension passed (0,001 s)
  - testGetChildren passed (0,002 s)
  - testGetCurrentWeightXY passed (0,002 s)
  - testGetCurrentWeights passed (0,001 s)
  - testGetAllReversals passed (0,002 s)
  - testGetAllReversalsXY passed (0,011 s)
  - testGetOPSC passed (0,001 s)
  - testGetTSC passed (0,005 s)
  - testGetSC passed (0,001 s)
  - testGetReversalProbability passed (0,001 s)
  - testGetWidthXY passed (0,002 s)
  - testGetHeightXY passed (0,001 s)
  - testGetWeights passed (0,029 s)
  - testGetXY passed (0,001 s)
  - testGetAlternativeCount passed (0,001 s)
  - testGetAlternativeDomain_int passed (0,007 s)
- com.man.priest.sensitivity.SimpleSATest passed
  - testSetCriterion passed (0,01 s)
  - testGetCriterion passed (0,0 s)
  - testGetRanking passed (0,008 s)
  - testGetRankReversals_0args passed (0,018 s)
  - testGetRankReversals_Criterion_int passed (0,0 s)

Figure B.3. Unit testing performed on PriEsT (continuation)

Appendix C
PriEsT Library Usage

In order to show how to use the library, the example presented in Section 2.6 can be solved using the library with the following code snippet:

```
/** Create problem and alternatives */
DecisionProblem problem = new DecisionProblem("Car");
problem.addAlternative("Avalon");
problem.addAlternative("Babylon");
problem.addAlternative("Carryon");

/** Add criteria and sub-criteria */
Criterion cPrice = problem.addCriteria("Price");
Criterion cMpg = problem.addCriteria("MPG");
Criterion cAmenities = problem.addCriteria("Amenities");
Criterion cPrestige = cAmenities.addCriterion("Prestige");
Criterion cComfort = cAmenities.addCriterion("Comfort");
Criterion cStyle = cAmenities.addCriterion("Style");

/** set PC matrix for each criterion */
PCMatrix m = new PCMatrix(3);
m.set(0, 1, 6);
m.set(0, 2, 3);
m.set(2, 1, 4);
problem.setPCMatrix(m);

m = new PCMatrix(3);
m.set(0, 1, 3);
m.set(0, 2, 6);
m.set(1, 2, 4);
cPrice.setPCMatrix(m);

/** ... and so on for each criterion */
cMpg.setPCMatrix(m);
cAmenities.setPCMatrix(m);
cPrestige.setPCMatrix(m);
cComfort.setPCMatrix(m);
cStyle.setPCMatrix(m);

/** Solve problem using Eigenvector */
Solver solver = new Solver();
AbstractMethod method = new EigenvectorMethod();
solver.addMethod(method);
solver.solve(problem);

/** Get ranking of alternatives */
double ranking[] = problem.getRanking(method);

/** Get weights of Price's children */
double weights[] = cPrice.getResult(method).get(0).getWeights();

/** Perform Probabilistic SA */
ProbabilisticSensitivityAnalyzer sa1;
sa1 = new ProbabilisticSensitivityAnalyzer(problem);
sa1.setMethod(method);
Statistics stats[] = sa1.simulate();
```

```
52
53 /** Perform Mathematical SA */
54 MathematicalSensitivityAnalyzer sa2;
55 sa2 = new MathematicalSensitivityAnalyzer(problem);
56 sa2.setMethod(method);
57 int ind[] = new int[] { 0, 1, 2 };
58 SensitivityDomain domain = sa2.analyze(problem.getRootNode(), ind);
59 double opsc = domain.getOPSC();
60
61 /** Feasible range for alternative 'Avalon' */
62 Range r1 = domain.getAlternativeDomain(0);
63
64 /** Polygon for alternative 'Avalon' */
65 List<Point2> points1 = r1.getPointsXY();
```

In line 1 the problem is created with “Car” as the name for the root node or goal. In lines 2-5 the alternatives are added to the problem, while in lines 7-13 the problem hierarchy is created by adding the criteria.

Lines 15-20 show how to create a PC matrix and assign it to a criterion. Lines 22-33 create the matrices for the rest of the criteria.

The problem is solved in lines 35-39 using the Eigenvector method. Additional methods can be added to the solver object using the same code as line 38.

The final ranking of alternatives is obtained in line 42. The value at index 0 of the array corresponds to the priority of the first alternative, “Avalon”. Index 1 and 2 are “Babylon” and “Carryon” respectively.

The weights of a specific criterion can be obtained as shown in line 45. As some methods produce more than one result, the “`.get(0)`” is necessary to get the first result.

Lines 47-51 show how to perform a probabilistic SA. By default all non-leaf nodes are analysed, as in this case. The results of the simulation can be obtained from the “`stats`” array, where each index of the array contains statistics for each alternative.

Lines 53-65 show how to perform a mathematical SA. Line 57 specifies the index of the children that will be analysed. Line 58 performs the analysis using the root node of the problem. That is, the SA is performed by changing the weights of “Price”, “MPG” and “Amenities” criteria.

Appendix D
Application Screen-shots

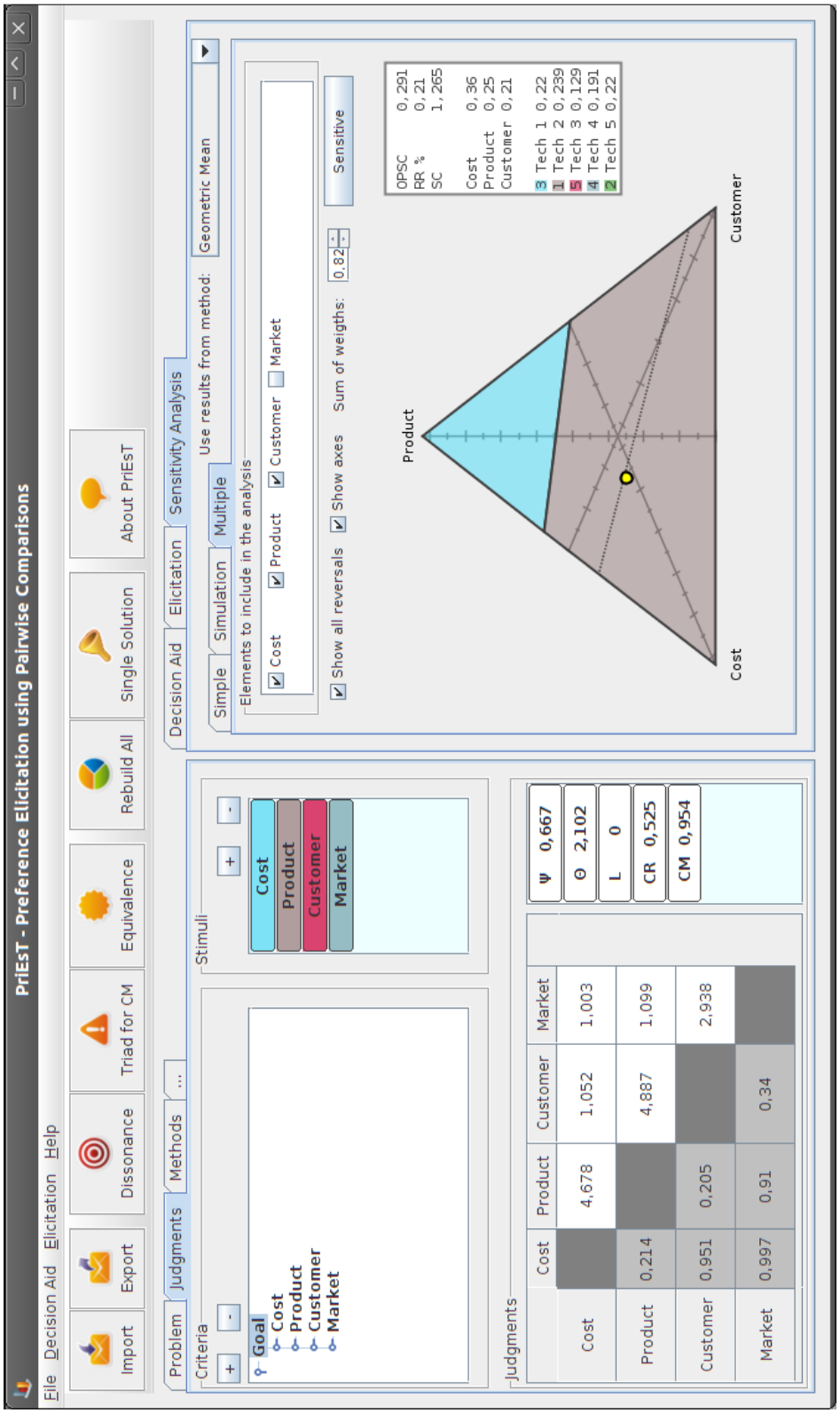

Figure D.1. PriEsT running on Ubuntu Linux

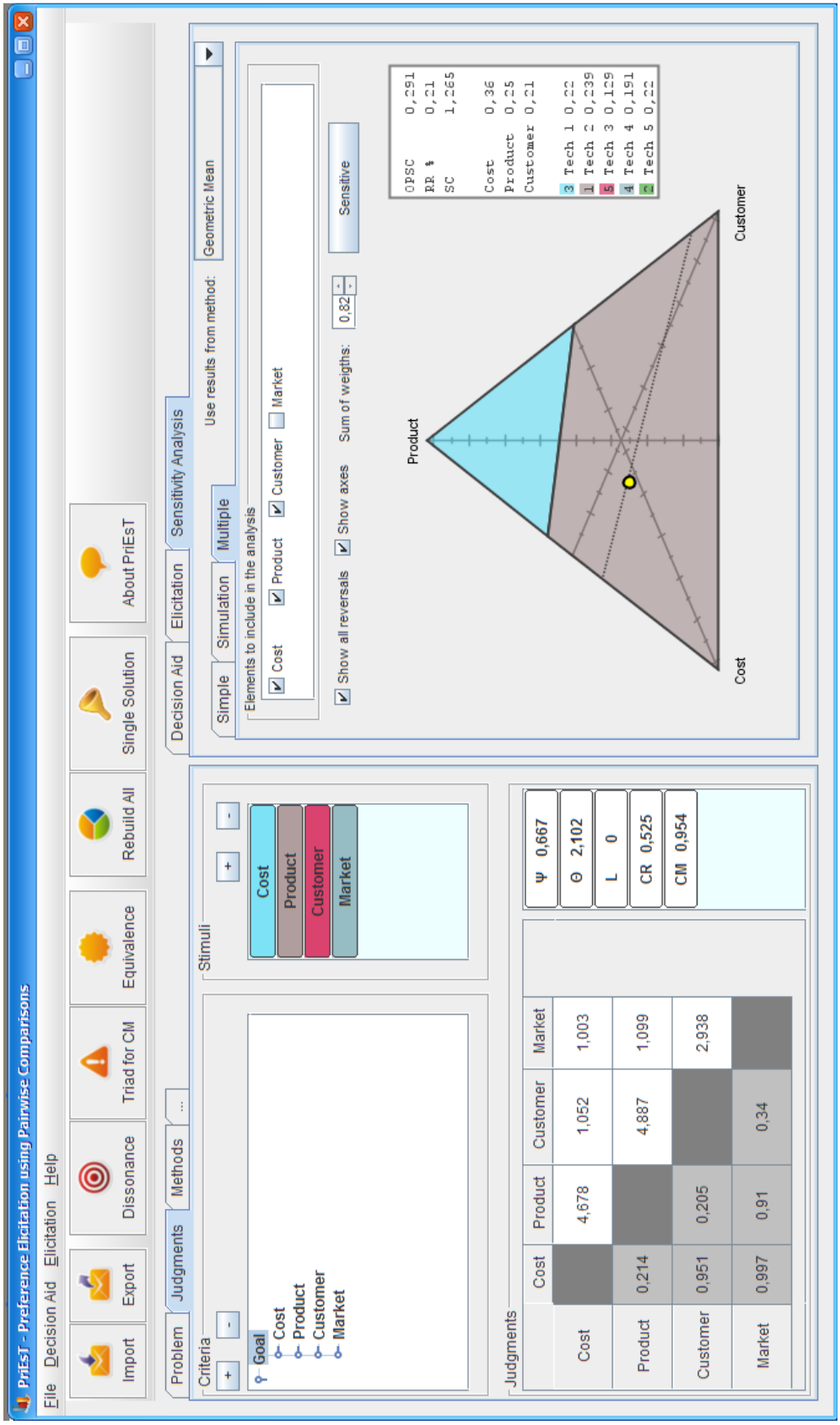

Figure D.2. PriEsT running on Windows XP

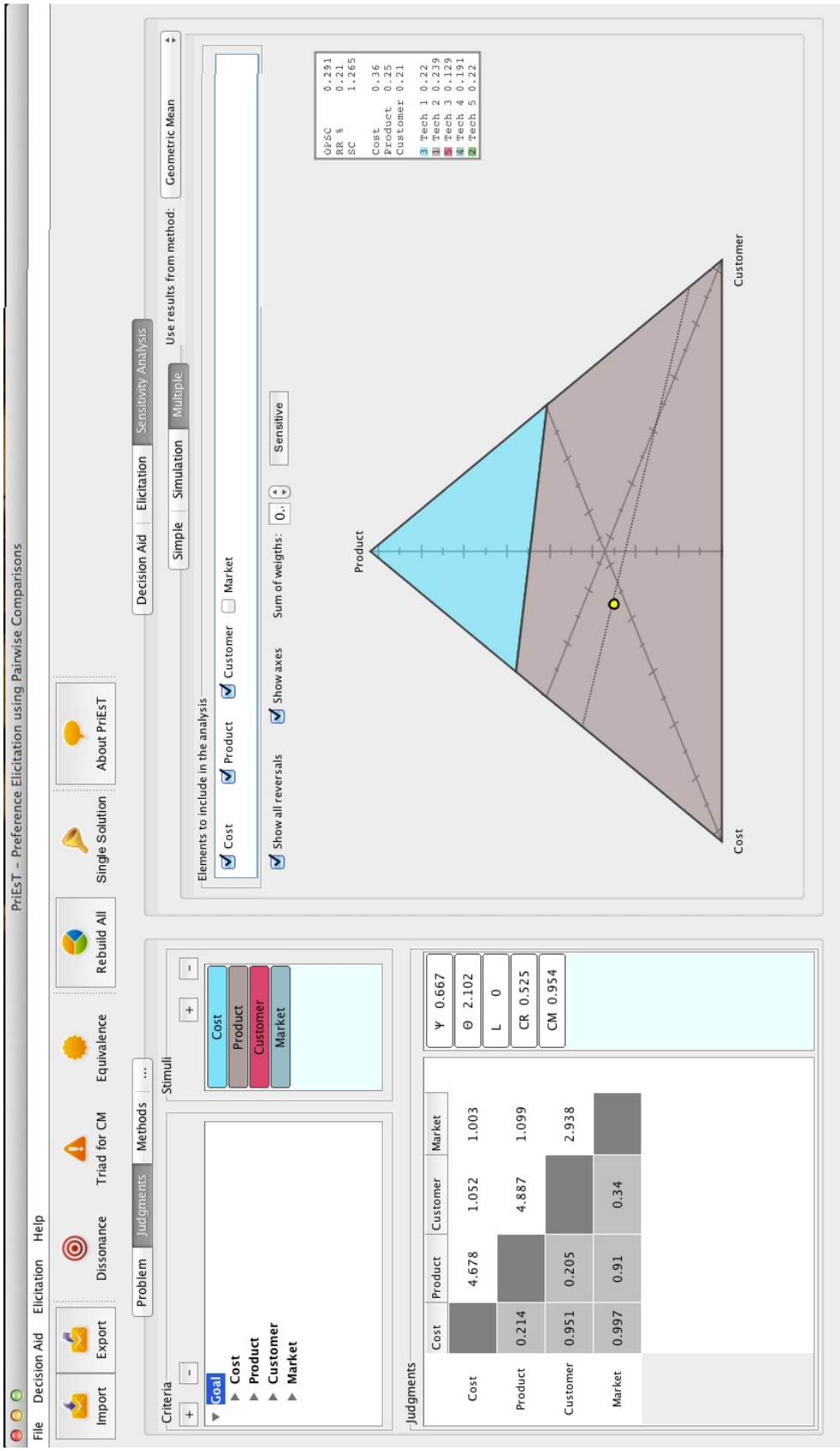

Figure D.3. PriEsT running on Mac OS X

Figure D.4. PriEsT Mobile running on Nokia 5800 - J2ME (left) and Samsung Galaxy S2 - Android (right)

Appendix E
Project Gantt Chart

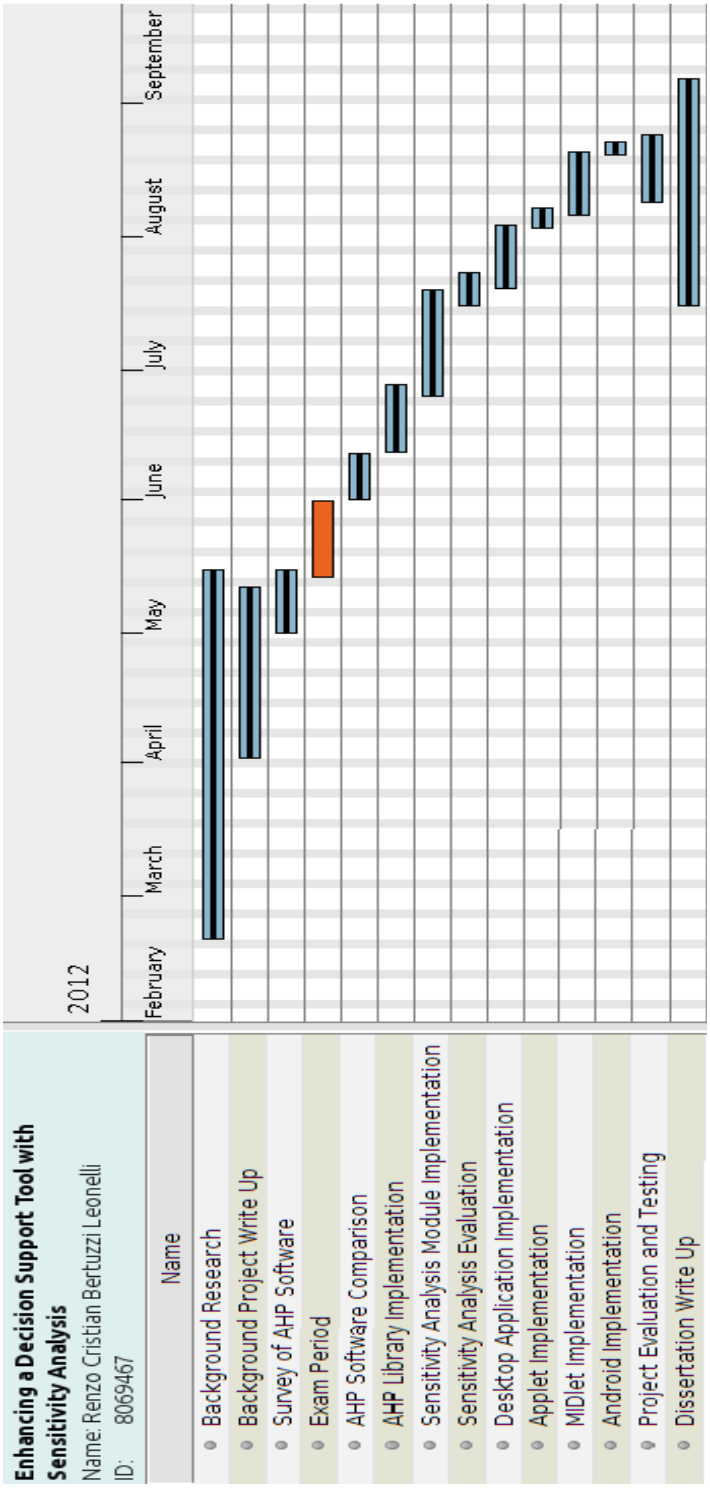

Figure E.1. Project Gantt Chart