An Overview of Mathematics for Students on a Joint Honours Programme

Andrea Schalk · June 13, 2013

How to use this document. This article was written for students on the joint honours CS and Maths programme in Manchester. It aims to give an overview of the field of mathematics, and discusses various areas students might wish to specialize in, pointing out which choices in Year 2 open up which areas in Year 3. Most students probably won't want to read the whole thing—read the overview and the curriculum section, and then pick those areas you want to know more about.[1]

1 Areas of Mathematics—an Overview

Traditionally, mathematics has been split into **pure** and **applied**. This division has never been as tidy as these labels suggest, but it will serve our purposes. In particular among applied subjects there are some areas that have been around for a long time, and consequently are quite large with several course units, and other areas that are more recent, and where there is typically only one course unit available. We here give an overview of these, and more details for specific parts are found in the 'Choices' section.

Applied maths. This is the mathematics of solving problems from various areas. Traditionally many of these come from physics, but you will also find problems from biology, computer science, or even how to practically solve problems arising from some areas of mathematics. How can we describe the forces that keep a structure standing? How can we describe the forces that apply in a liquid? How can we analyse data and draw conclusions from it? Under which circumstances can we calculate precise solutions for problems arising from these applications? How can we go about calculating approximate solutions when we can't find precise ones? How can we model networks and flow in those networks? What is the mathematics used to describe financial models? How can we use computers to solve problems from mathematics? Areas that have been around for a long time are concerned with partial differential equations, mechanics, viscous fluid, elasticity, and this also covers the areas of numerical mathematics and statistics. Areas where there is less tradition are mathematical programming, using computers to solve mathematical problems, mathematics of biology, and finance.

Pure maths. Course units in pure mathematics will typically develop a *mathematical theory*: Starting from some basic abstract definitions for the area, the lecturer will set out to explain the properties that follow from these definitions, and there will be theorems and their proofs involved. Some course units on pure mathematics manage to give practical work to the students which relies on carrying out calculations in particular structures (see for example the exam for Algebraic Structures 2), but nonetheless this is pure mathematics. Examples for course units in this area are: Algebraic Structures 1& 2, Topology, Group Theory, Geometry. Most logic course units will also fall into this category. Strictly speaking, the discipline of analysis (as the theory of limits) fits in here, but it also has close connections with applied mathematics and is the basis for many developments there as well.

Because they seek to answer different questions, course units from these two fields can be quite different in nature.

Course codes in mathematics split the area into fields. To find the field for a

- third year course unit, look at the *second digit*, for a
- second year unit, look at the *third digit*.[2]

[1] I would like to thank Francisco Lobo, Roman Krenicky and Joe Razavi for helping me improve these notes.

[2] The first digit in a course unit gives the year (1, 2, 3), the last digit whether it's first (1) or second semester (2), or both semesters (0). Second year units all have 0 as their second digit.

Here is a list of these fields, sorted by digit.

1. Pure Analysis

2. Pure Algebra

3. Logic

4. Applied Analysis—general concepts

5. Applied Analysis—PDE-based (including mechanics)

6. Applied Analysis—solving equations and numerical mathematics

7. Statistics—general concepts

8. Statistics—large data

9. Everything else (anything that doesn't fit into one of the traditional fields from above).

These digits are used to structure the 'Choices' section.

2 The Curriculum in Manchester

Students on the CM programme study a curriculum that takes them from A-level studies in the subject to advanced topics in the form of Level 3 course units. Students who wish to study the subject beyond that have the choice of various Masters degree programmes specializing in areas such as pure maths, applied maths, statistics or mathematical finance.

Year 1. Arguably, most of what happens in Year 1 of the CM degree is to bring all the students up to the same standard, with three of the four course units spending a lot of time covering material from the A-level syllabus (although some of it will only have been seen by students who have done Further Maths), and then developing that a bit further. The unit on *Sets, Functions and Proofs* is aiming to introduce some ideas which are important for pure mathematics in particular. For most students this is unfamiliar territory.

Year 2. This is where students encounter abstract mathematical theories for the first time. Compulsory course units are:

- MATH20201 Algebraic Structures 1. Many operations that appear variously have nice properties (if you're a mathematician) when considered from an abstract point of view (for example, the operations on a Rubik's cube, or on other geometric puzzles). This course unit looks at structures defined as a set with some operation(s) on it which have to satisfy various laws (associativity, commutativity, unit laws are typical examples). Once the basic definitions are in place, the course unit studies what can be deduced from them. This is the first view students get of a course in *pure mathematics*.

- MATH20111 Real Analysis. This is 'calculus done properly'. Starting from a formal definition of limit, one can formally define crucial concepts such as the continuity, differentiability and integrability of a function, and study under which circumstances these are well-defined. This is a basis for both, pure as well as applied, course units building on these ideas.

- MATH20142 Complex Analysis. The theory the field known as analysis changes considerably when it is based on the complex, rather than the real, numbers. Everything becomes much smoother and there are rather fewer awkward examples.

In Semester 1, students also have to pick one of

- MATH10141 Probability 1. This is a first year course unit which CM students may take in Year 2.

- MATH20411 Partial Differential Equations. This is required background for various course units in applied mathematics.
- MATH20701 Probability 2 This is only permitted for students who have taken **A-level Statistics**. It is the only way of specializing in statistics in Year 3.

Second semester options are discussed in the 'Choices' section.

Year 3. In the third year there are no compulsory course units. Students have to take 50 to 70 credits of maths (the remaining ones to 120 coming from computer science), and they have to do at least 40 credits of Level 3 material, but they may make up remaining credits with Level 2 material if they so wish. In mathematics there is a strict requirement for students to have taken the *prerequisites* for a course unit—hence picking the right optional units in Year 2 is vital to ensure students can pursue their chosen direction(s) in Year 3.

3 Choices for Specialization

Even single honours students of mathematics have to specialize, to some extent, during their final year. Since JH students, by definition, only do around half the number of course units of a single honours students, course unit choices in the *second year* are very important when it comes to opening up areas in the third year. Note that some lecturers only mention those prerequisites which are not compulsory for all maths students, and others include these—the latter practice makes it easier to judge which course units a course might be similar to.

Here is an overview of typical choices of specialization, and what students should pick in Year 2 to be equipped for their favourite area.[3]

3.1 Applied Maths

This is a big part of mathematics with very diverse areas. We use the 'grouping by course unit code' from above. The big areas here are **(applied) analysis (4–6)**, **numerical mathematics (6)**, **statistics (7,8)** and **other applied topics (9)**.

3.1.1 Applied Analysis: 4, 5, 6

This is a subject that tries to address problems that originate in physics or a related science. How can we mathematically describe the forces that apply in fluids, for example? What mathematical methods are useful for describing such situations, and how do they help us answer the resulting questions? These course units have not been popular with CM students in the past so I cannot say much about how CM students have done in them.

Students who are interested in this area, in particular course units with digit 6 should take MATH20411 Partial Differential Equations in Year 2, but note that some course units have **no formal prerequisites**.

- Digit 4. These typically require Real Analysis and go on to develop the subject further, but they look at concepts and techniques that are inspired by applications from physics. Some also build on Complex Analysis.
- Digit 5. Very specifically driven by wanting to model very specific situations, such as *viscous fluids*, *elastic materials*, *biology*, and *flow* (this includes applications coming from modelling any kind of transport phenomena). Many of these have **no formal prerequisites** and are

[3] This list of course units was correct for the academic year 2012/13—some changes will occur over time. Students should not assume that every course unit mentioned will be taught every year. Not all available course units are mentioned here, only ones most typical for their area and those most popular among CM students. You should still get an idea how other units fit into the classification scheme used here. Just because a course unit isn't mentioned doesn't mean you shouldn't pick it.

definitely worth considering for CM students. The one on biology, MATH35032, in particular might be worth looking at for students who want to avoid pure maths and heavy-duty applied maths. Students interested in this area might also want to look at the second year courses in MATH20502 Fluid Mechanics and MATH20512 Classical Mechanics.

- Digit 6. Possibly best described as a less application-driven approach to the mathematical structures that appear in the above applications; so one might, for example, study what solutions there are for particular PDEs. Note that I treat Numerical Mathematics separately below.

  In the past MATH36032 Problem Solving by Computer has been popular with CM students. This is a course unit which is assessed entirely through coursework, where Matlab is used to solve various problems. Note that numbers on this course units are *restricted* and that it is compulsory for some students, so places are hard to get.

3.1.2 Numerical Mathematics: 6

This is the discipline that aims to solve problems *approximately*, typically because there is no known way of calculating a precise solution, or because that is too costly computationally. The discipline looks at algorithms for calculating such approximate solutions, estimating the error of the computation at each stage, and looking into how such errors propagate. In Manchester, only **Numerical Analysis** is taught at up to Level 3. Students who wish to pursue this direction have to take

- MATH20411 Partial Differential Equations and
- MATH20602 Numerical Analysis 1

in Year 2 and

- MATH36022 Numerical Analysis 2

in Year 3.

3.1.3 Probability and Statistics: 7, 8

Being on a joint honours course unit makes it very difficult to study this area in any depth.

Students who have taken A-level Statistics may take

- MATH20701 Probability 2 and
- MATH20802 Statistical Methods

in Year 2 which allows them to pick from a fairly large selection of course units in Year 3 (note that some of these also require MATH20812 Practical Statistics).

Students who have not taken A-level Statistics cannot progress to most of the Level 3 material, they are restricted to taking

- MATH10141 Probability 1 in Year 2 and
- MATH20701 Probability 2,
- MATH20802 Statistical Methods
- MATH38152 Social Statistics in Year 3.

3.1.4 Other applied mathematics: 9

The applicability of mathematics is not limited to those areas which are traditionally considered 'applied'. Specifically those students who do not particularly want to specialize in any of the areas suggested above there are sufficiently many course units remaining to cover their third year.

In the second year, there are the following two course units on offer.

- MATH20902 Discrete Mathematics
- MATH20912 Introduction to Financial Mathematics.

However, none of these are prerequisites for third year units, and it might be prudent to wait until third year to take these. That allows them to take other course units that open up more choice during Year 3.

The following course units are on offer in this area at Level 3.

- MATH39001 Combinatorics and Graph Theory. Combinatorics is about being able to count all combinations of something under certain conditions (eg, how many ways are there of forming couples at a ball?), while graph theory has applications in modelling networks, connectivity, and similar structures.
- MATH39012 Mathematical Programming. Linear programming, an area that covers the simplex algorithm and related material.
- MATH39032 Mathematical Modelling in Finance. Does not have the second year unit as a prerequisite, but I've been told it's helpful to have done that one.

Other course units students might want to look at are:

- MATH32031 Coding Theory. Pure maths course unit, but has been popular with CM students in the past, and is somewhat application-oriented (see also Section 3.2.2).
- MATH35032 Mathematical Biology. Starts with using ODEs and PDEs, but then looks at applying other mathematical tools to problems from biology.

3.2 Pure Maths

This subject area may be split further into three areas, namely **algebra**, **analysis**, and **logic**. It is important to stress that pure mathematics is the rigorous development of properties (in the form of theorems and their proofs) from formal definitions—so students should be prepared to cover a fair selection of theorems and their proofs. However, some time may also be devoted to studying particular example structures, so it is possible that such a course unit may contain some computations of the form you may feel more familiar with from A-level maths. Students who want the widest range of pure maths topics available to them in Year 3 should take

- MATH20212 Algebraic Structures 2 and
- MATH20122 Metric Spaces

in the second semester of Year 2.

3.2.1 Analysis: 1

This area builds on the Real Analysis course unit from Year 2, but takes it into a fairly wide variety of directions. Depending on which directions students want to take, they will have to study

- MATH20122 Metric Spaces (we can defined notions of limits, convergence, continuity, etc, provided we have a notion of distance between two points)

or

- MATH20222 Introduction to Geometry (connects notions from Linear Algebra like vectors, their products, algebra—curves and surfaces described as solutions to sets of equations—and Real Analysis—velocity, acceleration, curvature—to study $\mathbb{R}^2$ and $\mathbb{R}^3$) and/or
- MATH20132 Calculus of Several Variables (Analysis, but now with functions that take more than one variable)

in the second semester of Year 2. This area hasn't been popular with CM students in the past, possibly because it requires particular choices in Year 2. Here's just a glimpse of the Level 3 topics available:

- MATH31011 Fourier Analysis and Lebesgue Integration. Is there a way of defining integrals other than that you saw in Real Analysis? Very abstract.
- MATH31051 Introduction to Topology. This is analysis without numbers (no ϵ/δ arguments). It asks whether we can define key notions such as convergence and continuity without having a notion of distance. A more abstract view of what is covered in MATH20122 Metric Spaces.

There are more options, such as MATH31002 Linear Analysis (spaces with infinite dimensions, building on MATH20122 Metric Spaces) and MATH31022 Analytic Number Theory (use MATH20142 Complex Analysis to study the integers, including prime numbers and functions on integers), and additional options requiring students to have studied both, MATH20222 Introduction to Geometry and MATH20132 Calculus of Several Variables in Year 2.

3.2.2 Algebra: 2

Students wishing to study algebra in Year 3 *must* take

- MATH20212 Algebraic Structures 2

in the second semester of Year 2. This then allows them to pick, for example, the following course units in Year 3:

- MATH32001 Group Theory. This goes into more detail into the theory of groups. It's fairly abstract.
- MATH32031 Coding Theory. This is about efficiently encoding messages, *not* about encrypting and decrypting secrets. It'll show you what you can do with some of the more general ideas from linear algebra. Note that this course unit requires only Algebraic Structures 1 as a prerequisite, and is probably the least intimidating pure maths course unit. Could be viewed as being a non-traditional applied unit.
- MATH32051 Hyperbolic Geometry. This unit addresses the question how one might address axiomatizing the geometry of the plain, and beyond.
- MATH39001 Combinatorics and Graph Theory. Not treated as a pure maths course units in the syllabus, and has no prerequisites, but in general fits into the idea of structures and their properties, but here taught with a more applied flavour.

There are other course units that fit into this framework, which have been popular with CM students in the past, such as MATH32012 Commutative Algebra (more structures along the lines of Algebraic Structures 2) and MATH32062 Introduction to Algebraic Geometry (this is about surfaces defined in $\mathbb{R}^n$). I'd also suggest students might want to look at MATH31051 Introduction to Topology, discussed in more detail in Section 3.2.1.

3.2.3 Logic: 3

This is the study of formal systems consisting of formulae together with rules for deriving them, and said rules typically are considered formalisations of valid reasoning principles. While Logic is a subject that can be studied in-depth in Manchester at MSc level, course units for undergraduates are in limited supply. In Year 2 we have

- MATH20302 Propositional Logic (How can we formalise mathematical arguments, and what notions of such arguments being true are there? How do they relate?)

and on Level 3

- MATH33001 Predicate Logic (extends the previous course by allowing more complicated formulae to be built). Note that this does not have the Level 2 unit as a prerequisite, although it will be easier to understand for students who have that background.